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ECS 17: Data, Logic, and Computing
Final
March 19, 2024

Notes:

1) The final is open book, open notes.
2) You have 2 hours, no more: I will strictly enforce this.
3) The final is graded over 100 points
4) Please, check your work! **Also, do show your work**

Part I Data (10 questions, each 3 points; total 30 points)
(These questions are multiple choices; in each case, find the most **plausible** answer)

1) Which binary number is $(101111)_2 + 1$ equal to?

a. $(101112)_2$
b. $(101110)_2$
c. $(111111)_2$
d. $(110000)_2$

2) How much space would you need to store a 10 min song that has been sampled at 44.1 KHz, with each data point stored on 8 bits, in quadrophony (i.e. using 4 microphones to record the song; assume no compression)?

a. About 100 MBytes
b. About 100 Mbits
c. About 50 Mbytes
d. About 5 Mbytes

3) $(1101)_2 - (110)_2 =$

a. #B
b. #7
c. #A
d. #6

4) #C9-#B3=

a. $(10110)_2$,
b. $(11110)_2$,
c. $(10111)_2$,
d. $(11111)_2$.

5) Assuming that there are 400,000 characters in the UNICODE, what is the minimal number of bits needed to store a word of 8 characters with this code?

a. 19,
b. 190,
c. 152,
d. 148.

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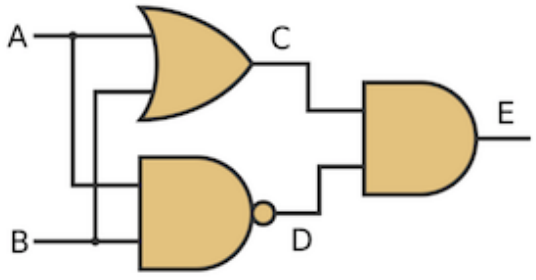
6) ***If a signal is thought to have a maximum frequency between 1000 Hz and 4000 Hz, which of the following would be the most appropriate sample rate?***
   a. 500 Hz,
   b. 4000 Hz,
   c. 8000 Hz,
   d. 9000 Hz.

7) ***As you are reading a paper, you see the word LOA_ (upper case), where the underscore is unfortunately a letter you cannot read. You know, however, that the sum of all ASCII identifiers for this word is #120. What is the missing letter represented as the underscore***
   a. S,
   b. F,
   c. D,
   d. N.

8) ***Let A be the hexadecimal number #F1 and B the hexadecimal number #101; which of these hexadecimal numbers C satisfies A+C = B?***
   a. #A0,
   b. #10,
   c. #11,
   d. #1F2.

9) ***Mannikins are birds that can flap their wings up to 100 times a second. Which of these sampling rates is most appropriate to use if you want to monitor the flight of a mannikin correctly with a digital device?***
   a. 3 Hz,
   b. 30 Hz,
   c. 300 Hz,
   d. 100 Hz.

10) ***The gate shown below is equivalent to:***



   a. The AND gate,
   b. The OR gate,
   c. The XOR gate,
   d. The XNOR gate.

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Part II Logic (three problems; total 30 points)

1) For each of the five propositions in the table below, indicates on the right if they are always **tautologies or not** (*x* is a real number, *p* and *q* are propositions, T is a tautology, and F a contradiction) ***(10 points)***.

Proposition	**Tautology (Yes or No)**
If $x^4 = -x^2 - 1$ then 25=23+3	
$(p \wedge \neg p) \rightarrow q$	
$(p \vee F) \rightarrow p$	
$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$	
$(p \wedge \neg q) \vee (\neg p \wedge q)$	

2) Inspector Craig from Scotland Yard has been assigned a special mission: identify a knave on an island that otherwise is inhabited only by knights. This island, however, is unusual: knights always tell the truth in the morning and always lie in the afternoon, while knaves always lie in the morning and tell the truth in the afternoon. As he arrives on the island, he is presented with 3 inhabitants, Alice, Ben, and Claire. He is told that one of them is the knave he is looking for (the two others are then knights). Unfortunately, he does not know if it is currently morning or afternoon. Alice tells him, "if I am a knave, it is currently morning", while Claire tells him that "Alice is a knave". Can you help Inspector Craig find the knave? Can you also tell him if it is morning or afternoon? ***(10 points)***

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1) 3) Let p and q be two propositions. Use a truth table or logical equivalence to show that the proposition $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ is a tautology. ***(10 points)***

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Part III. Proofs (4 questions; each 10 points; total 40 points)

1) Let *n* be an integer. Show that *n* is odd if and only if $n^2 - 1$ is a multiple of 8 (note that an integer *n* is a multiple of 8 if and only if there exists an integer *k* such that $n = 8k$).

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2) Use induction to prove that any postage value of *n* cents can be made with only 5-cent stamps and 7-cent stamps, whenever $n \geq 24$, *n* natural number.

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3) Let u_n be the sequence of real numbers defined by:

$$u_0 = 3$$

$$u_{n+1} = \frac{u_n - 2}{2u_n + 5}$$

We will assume that $u_n \neq \frac{-5}{2}$, i.e. that the sequence is defined for all natural numbers *n*. Show that for all natural number *n*,

$$u_n = \frac{9 - 8n}{3 + 8n}$$

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4) Let f_n be the n-th Fibonacci number (note: Fibonacci numbers satisfy $f_0=0, f_1=1$ and $f_n + f_{n+1} = f_{n+2}$). Prove by induction that for all natural numbers $n \geq 1$, f_{3n} *is even.*

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Appendix A: ASCII table

Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char
0	00	Null	32	20	Space	64	40	@	96	60	`
1	01	Start of heading	33	21	!	65	41	A	97	61	a
2	02	Start of text	34	22	"	66	42	B	98	62	b
3	03	End of text	35	23	#	67	43	C	99	63	c
4	04	End of transmit	36	24	$	68	44	D	100	64	d
5	05	Enquiry	37	25	%	69	45	E	101	65	e
6	06	Acknowledge	38	26	&	70	46	F	102	66	f
7	07	Audible bell	39	27	'	71	47	G	103	67	g
8	08	Backspace	40	28	(	72	48	H	104	68	h
9	09	Horizontal tab	41	29	)	73	49	I	105	69	i
10	0A	Line feed	42	2A	*	74	4A	J	106	6A	j
11	0B	Vertical tab	43	2B	+	75	4B	K	107	6B	k
12	0C	Form feed	44	2C	,	76	4C	L	108	6C	l
13	0D	Carriage return	45	2D	-	77	4D	M	109	6D	m
14	0E	Shift out	46	2E	.	78	4E	N	110	6E	n
15	0F	Shift in	47	2F	/	79	4F	O	111	6F	o
16	10	Data link escape	48	30	0	80	50	P	112	70	p
17	11	Device control 1	49	31	1	81	51	Q	113	71	q
18	12	Device control 2	50	32	2	82	52	R	114	72	r
19	13	Device control 3	51	33	3	83	53	S	115	73	s
20	14	Device control 4	52	34	4	84	54	T	116	74	t
21	15	Neg. acknowledge	53	35	5	85	55	U	117	75	u
22	16	Synchronous idle	54	36	6	86	56	V	118	76	v
23	17	End trans. block	55	37	7	87	57	W	119	77	w
24	18	Cancel	56	38	8	88	58	X	120	78	x
25	19	End of medium	57	39	9	89	59	Y	121	79	y
26	1A	Substitution	58	3A	:	90	5A	Z	122	7A	z
27	1B	Escape	59	3B	;	91	5B	[	123	7B	{
28	1C	File separator	60	3C	<	92	5C	\	124	7C	|
29	1D	Group separator	61	3D	=	93	5D	]	125	7D	}
30	1E	Record separator	62	3E	>	94	5E	^	126	7E	~
31	1F	Unit separator	63	3F	?	95	5F	_	127	7F	□

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Appendix B: Binary to Hexadecimal

Base 10	Base 2	Base 16
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

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Appendix C

The ECS17 Prayers

1) Thou shalt not say "there exists *k*" without mentioning the domain of *k*.
2) Thou shalt not say "it is obvious"
3) If *p* and *q* are two propositions, then $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$. This is the basis for the proof by contrapositive.
4) If *p* and *q* are two propositions, then $p \rightarrow q \Leftrightarrow \neg p \vee q$. This is the basis for the proof by contradiction.
5) An integer *n* is even if and only if there exists and integer *k* such that $n = 2k$. We say also that n is a multiple of 2.
6) An integer *n* is odd if and only if there exists and integer *k* such that $n = 2k + 1$.
7) BEWARE of divisions and square roots when you are working with integers.

Proofs that you can use without proving them again

We can use the following results without having to validate them:

1) Let *n* be an integer. Then:
   a) If *n* is even, then *n+1* and *n-1* are odd
   b) if *n* is odd, then *n+1* and *n-1* are even
2) Let *n* be an integer. Then:
   a) *n* is even, if and only if n^2 is even
   b) *n* is odd, if and only if n^2 is odd
3) $\forall n \in Z, n(n+1)$ is even.
4) $\sqrt{2}$ is irrational.

Identities

Let *a* and *b* be two real numbers:

1) $(a+b)^2 = a^2 + 2ab + b^2$
2) $(a-b)^2 = a^2 - 2ab + b^2$
3) $a^2 - b^2 = (a-b)(a+b)$
4) **Completing the square:** $a^2 + b^2 = (a+b)^2 - 2ab$