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ECS 17: Data, Logic, and Computing
Final
March 18, 2025

Notes:

1) The final is open book, open notes.
2) You have 2 hours, no more: I will strictly enforce this.
3) The final is graded over 100 points
4) Please, check your work! **Also, do show your work**

Part I Data (10 questions, each 3 points; total 30 points)
(These questions are multiple choices; in each case, find the most **plausible** answer)

1) ***Convert the binary number $(1011010111001100)_2$ to hexadecimal:***
   a. #B5CC
   b. #B6CC
   c. #D6CC
   d. #D5CC

2) ***What is the largest signed integer that can be stored in 4 bits?***
   a. 15
   b. 16
   c. 7
   d. 8

3) ***$(1101)_2 + (1011)_2 =$***
   a. #18
   b. #1A
   c. #1C
   d. #1E

4) ***A camera captures video at 24 frames per second. Which of these moving objects can be properly recorded (Select all that apply)***
   a. A hummingbird's wings flapping at 50 Hz
   b. A ceiling fan rotating at 0.8 revolutions per second
   c. A metronome moving at 30 beats per minute
   d. A strobe light flashing 10 times per second

5) ***A 1-minute video is captured with 30 frames per second, with each frame having a resolution of 1920×1080 pixels, and each pixel stored over 24-bit. What is the uncompressed file size in gigabytes (GB)? (Assume 1 GB = 10^9 bytes)***
   a. 8.9 GB
   b. 11.2 GB
   c. 3.7 GB
   d. 5.6 GB

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6) ***Which binary number comes right before the binary number 1000?***
   a. 0999
   b. 0111
   c. 0100
   d. 1001

7) ***Decode the name whose ASCII representation is #54 #68 #6F #6D #61 #73***
   a. Thomas
   b. THOMAS
   c. Thomes
   d. THOMES

8) ***Let A be the number with the hexadecimal representation #A and B the number whose hexadecimal representation is #18; which of these numbers X (in hexadecimal form) satisfies $X^2 - AX + B = 0$ (circle all that apply):***
   a. #C
   b. #4
   c. #2
   d. #6

9) ***What time is it on this digital clock (dark circle mean "on")?***

   a. 21:24:16
   b. 21:24:17
   c. 21:34:16
   d. 21.34.17

10) ***Let x be a hexadecimal digit (i.e. a number in {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}. We know that #x3 + #2x = #BC (where # means that the numbers are given in hexadecimal format; for example, if x = 1, #x3 = #13 = $(19)_{10}$). Solve for x:***

   a. 7
   b. 8
   c. 9
   d. A

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Part II Logic (three problems; total 30 points)

1) For each of the five propositions in the table below, indicates on the right if they are always **tautologies or not** (p and q are propositions) ***(10 points)***.

Proposition	Tautology (Yes or No)
$p \rightarrow p$	
$(p \vee q) \rightarrow (p \wedge q)$	
$(p \wedge q) \rightarrow (p \vee q)$	
if 3+6 = 10 then 25=16+10	
$(p \wedge \neg q) \vee (\neg p \vee q)$	

2) Let p and q be two propositions. Use a truth table or logical equivalence to show that the proposition $[\neg(p \rightarrow q)] \leftrightarrow [p \wedge \neg q]$ is a tautology. ***(10 points)***

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3) Inspector Craig from Scotland Yard has been assigned a special mission: find a treasure supposedly hidden on an island where there is a single village. This village is inhabited only by truth-tellers and liars. Truth-tellers always tell the truth, while liars always lie. Inspector Craig meet two villagers, **Daniel** and **Eve**, and ask them about the treasure. **Daniel** says: "If I am a truth-teller, then the treasure is hidden in my house" while **Eve** says: "Daniel is a liar."
Can you help inspector Craig and tell him if the treasure is in Daniel's house? Can you determine whether Daniel and Eve are truth-tellers or liars? Justify your answer. ***(10 points)***

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Part III. Proofs (4 questions; each 10 points; total 40 points)

1) Give a ***direct proof*** and an ***indirect proof*** of the proposition: if $2n^5 + n^2+4n+2$ is odd, then n is odd, where n is a natural number.

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2) Use induction to prove that any postage value of n cents can be made with only 4-cent stamps and 7-cent stamps, whenever $n \geq 18$, n natural number.

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3) Let F_n be the n-th Fibonacci number (note: Fibonacci numbers satisfy $F_0=0$, $F_1=1$ and $F_n + F_{n+1} = F_{n+2}$ for $n \geq 0$). Prove by induction that for all natural numbers $n \geq 0$, F_{5n} is a multiple of 5 (*where "multiple of 5" means that there exits an integer m such that* $F_{5n} = 5m$).

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4) Let a_n be the sequence of integers defined by $a_n = 3a_{n-1} + 2$ for *n≥2*, with $a_1 = 5$. Show using a proof by induction that $a_n - 1$ is divisible by 4, for all natural numbers *n*. (*Reminder:* $a_n - 1$ *is divisible by 4 means that there exits an integer k such that* $a_n - 1 = 4k$)

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Appendix A: ASCII table

Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char
0	00	Null	32	20	Space	64	40	@	96	60	`
1	01	Start of heading	33	21	!	65	41	A	97	61	a
2	02	Start of text	34	22	"	66	42	B	98	62	b
3	03	End of text	35	23	#	67	43	C	99	63	c
4	04	End of transmit	36	24	$	68	44	D	100	64	d
5	05	Enquiry	37	25	%	69	45	E	101	65	e
6	06	Acknowledge	38	26	&	70	46	F	102	66	f
7	07	Audible bell	39	27	'	71	47	G	103	67	g
8	08	Backspace	40	28	(	72	48	H	104	68	h
9	09	Horizontal tab	41	29	)	73	49	I	105	69	i
10	0A	Line feed	42	2A	*	74	4A	J	106	6A	j
11	0B	Vertical tab	43	2B	+	75	4B	K	107	6B	k
12	0C	Form feed	44	2C	,	76	4C	L	108	6C	l
13	0D	Carriage return	45	2D	-	77	4D	M	109	6D	m
14	0E	Shift out	46	2E	.	78	4E	N	110	6E	n
15	0F	Shift in	47	2F	/	79	4F	O	111	6F	o
16	10	Data link escape	48	30	0	80	50	P	112	70	p
17	11	Device control 1	49	31	1	81	51	Q	113	71	q
18	12	Device control 2	50	32	2	82	52	R	114	72	r
19	13	Device control 3	51	33	3	83	53	S	115	73	s
20	14	Device control 4	52	34	4	84	54	T	116	74	t
21	15	Neg. acknowledge	53	35	5	85	55	U	117	75	u
22	16	Synchronous idle	54	36	6	86	56	V	118	76	v
23	17	End trans. block	55	37	7	87	57	W	119	77	w
24	18	Cancel	56	38	8	88	58	X	120	78	x
25	19	End of medium	57	39	9	89	59	Y	121	79	y
26	1A	Substitution	58	3A	:	90	5A	Z	122	7A	z
27	1B	Escape	59	3B	;	91	5B	[	123	7B	{
28	1C	File separator	60	3C	<	92	5C	\	124	7C	|
29	1D	Group separator	61	3D	=	93	5D	]	125	7D	}
30	1E	Record separator	62	3E	>	94	5E	^	126	7E	~
31	1F	Unit separator	63	3F	?	95	5F	_	127	7F	□

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Appendix B: Binary to Hexadecimal

Base 10	Base 2	Base 16
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

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Appendix C

The ECS17 Potato Prayers

1) Thou shalt not say "there exists k" without mentioning the domain of k.
2) Thou shalt not say "it is obvious"
3) If p and q are two propositions, then $p \to q \Leftrightarrow \neg q \to \neg p$. This is the basis for the proof by contrapositive.
3) If p and q are two propositions, then $p \to q \Leftrightarrow \neg p \lor q$. This is the basis for the proof by contradiction.
4) An integer n is even if and only if there exists and integer k such that $n = 2k$. We say also that n is a multiple of 2.
5) An integer n is odd if and only if there exists and integer k such that $n = 2k + 1$.
6) BEWARE of divisions and square roots when you are working with integers.

Proofs that you can use without proving them again

We can use the following results without having to validate them:

1) Let n be an integer. Then:
   a) If n is even, then $n + 1$ is odd
   b) if n is odd, then $n + 1$ is even
2) Let n be an integer. Then:
   a) n is even, if and only if n^2 is even
   b) n is odd, if and only if n^2 is odd
3) $\forall n \in \mathbb{Z}, \quad n(n+1)$ is even.
4) $\sqrt{2}$ is irrational.