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ECS 17: Data, Logic, and Computing
Final
March 16, 2026

Notes:

- 1) The final is open book, open notes.
- 2) You have 2 hours, no more: I will strictly enforce this.
- 3) The final is graded over 100 points
- 4) Please, check your work! **Also, do show your work**

Part I Data (10 questions, each 3 points; total 30 points)

(These questions are multiple choices; in each case, find the most **plausible** answer)

1) $(11111)_2 + \#AE$:

- a. $(205)_{10}$
- b. $(206)_{10}$
- c. $(204)_{10}$
- d. $(203)_{10}$

2) *What is the largest unsigned integer that can be stored in 3 bits?*

- a. 7
- b. 8
- c. 15
- d. 16

3) $(88)_{10} + (10111)_2 =$

- a. #6F
- b. #6E
- c. #E6
- d. #F6

4) *A smartphone camera is set to record slow-motion video at 60 frames per second. Which of the following continuous mechanical motions will be captured accurately without aliasing? (Select all that apply)*

- a. A car engine idling at 3000 RPM (Revolutions Per Minute)
- b. A DJ's vinyl record spinning at 45 RPM
- c. A power drill rotating at 15 Revolutions per second
- d. An acoustic guitar's "A" string vibrating at 110 Hz

5) *A 1-minute video is captured with 30 frames per second, with each frame having a resolution of 1080×1080 pixels. How much space is used for each pixel if the video occupies approximately 4.2 gigabytes (GB)? (Assume $1 \text{ GB} = 10^9$ bytes)*

- a. 8 bits
- b. 16 bits
- c. 32 bits
- d. 24 bits

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- 6) *What is the binary representation of the number that is half the binary number $(1000)_2$?*
- a. $(0500)_2$
 - b. $(0100)_2$
 - c. $(0010)_2$
 - d. $(0004)_2$
- 7) *The sum of the ASCII codes for the word ECS17 is:*
- a. #143
 - b. #144
 - c. #145
 - d. #142
- 8) *Let n be the number with the hexadecimal representation #A and m the number whose hexadecimal representation is #18; which of these numbers X (in hexadecimal form) satisfies $X^2 + nX - m = 0$ (circle all that apply):*
- a. #D
 - b. #4
 - c. #2
 - d. #E
- 9) *A biologist wants to film a bee's wings flapping at 230 Hz and a moth's wings flapping at 40 Hz so that neither insect experiences the "wagon-wheel effect" on camera. Which of the following camera frame rates would successfully capture both? (Select all that apply)?*
- a. 24 frame per seconds (fps)
 - b. 60 fps
 - c. 240 fps
 - d. 500 fps
- 10) *Let x be a hexadecimal digit (i.e. a number in $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}$). We know that $\#xA + \#Fx = \#1A4$ (where # means that the numbers are given in hexadecimal format; for example, if $x = 1$, $\#xA = \#1A = (26)_{10}$). Solve for x :*
- a. 7
 - b. A
 - c. B
 - d. C

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Part II Logic (three problems; total 30 points)

1) For each of the five propositions in the table below, indicates on the right if they are always **tautologies or not** (n is an integer, p and q are propositions) (**10 points**).

Proposition	Tautology (Yes or No)
$p \rightarrow \neg p$	
$(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$	
$(p \wedge q) \rightarrow (p \vee q)$	
if $n^2 + n$ is odd then $n=5$	
$(p \wedge \neg p) \vee (\neg q \vee q)$	

2) Let p and q be two propositions. Use a truth table or logical equivalence to show that the proposition $[(p \rightarrow q) \rightarrow p] \rightarrow p$ is a tautology. (**10 points**)

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3) Commander Ripley is exploring a derelict space station trying to locate the vital "override key." The station's computer system has rebooted, and now its android crew is split into two factions: Security Droids and Corrupted Droids. Security Droids always tell the truth, while Corrupted Droids always lie. Ripley encounters two androids, Alpha and Beta.

- **Alpha says:** "If I am a Security Droid, then the override key is in the Reactor Core."
- **Beta says:** "Alpha is a Corrupted Droid."

Can you help Commander Ripley determine if the override key is in the Reactor Core? Can you figure out whether Alpha and Beta are Security Droids or Corrupted Droids? Justify your answer. *(10 points)*

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Part III. Proofs (4 questions; each 10 points; total 40 points)

1) Use a **proof by induction** to show that $3^n - 1$ is even, for all natural numbers n .

2) Let F_n be the n -th Fibonacci number (note: Fibonacci numbers satisfy $F_0=0$, $F_1=1$ and $F_n + F_{n+1} = F_{n+2}$ for $n \geq 0$). Prove by induction that for all natural numbers $n \geq 0$, F_{4n} is a multiple of 3 (where “multiple of 3” means that there exists an integer m such that $F_{4n} = 3m$).

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3) Use induction to prove that any postage value of n cents can be made with only 3-cent stamps and 8-cent stamps, whenever $n \geq 15$, n natural number.

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- 4) Let n be a natural number and a_n be the sequence of real numbers defined by

$$a_{n+1} = \frac{a_n}{\sqrt{a_n^2 + 1}} \text{ for } n \geq 1, \text{ with } a_1 = 1.$$

Show using a proof by induction that $a_n = \frac{1}{\sqrt{n}}$ for all natural numbers n .

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Appendix A: ASCII table

Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char
0	00	Null	32	20	Space	64	40	@	96	60	`
1	01	Start of heading	33	21	!	65	41	A	97	61	a
2	02	Start of text	34	22	"	66	42	B	98	62	b
3	03	End of text	35	23	#	67	43	C	99	63	c
4	04	End of transmit	36	24	\$	68	44	D	100	64	d
5	05	Enquiry	37	25	%	69	45	E	101	65	e
6	06	Acknowledge	38	26	&	70	46	F	102	66	f
7	07	Audible bell	39	27	'	71	47	G	103	67	g
8	08	Backspace	40	28	(72	48	H	104	68	h
9	09	Horizontal tab	41	29)	73	49	I	105	69	i
10	0A	Line feed	42	2A	*	74	4A	J	106	6A	j
11	0B	Vertical tab	43	2B	+	75	4B	K	107	6B	k
12	0C	Form feed	44	2C	,	76	4C	L	108	6C	l
13	0D	Carriage return	45	2D	-	77	4D	M	109	6D	m
14	0E	Shift out	46	2E	.	78	4E	N	110	6E	n
15	0F	Shift in	47	2F	/	79	4F	O	111	6F	o
16	10	Data link escape	48	30	0	80	50	P	112	70	p
17	11	Device control 1	49	31	1	81	51	Q	113	71	q
18	12	Device control 2	50	32	2	82	52	R	114	72	r
19	13	Device control 3	51	33	3	83	53	S	115	73	s
20	14	Device control 4	52	34	4	84	54	T	116	74	t
21	15	Neg. acknowledge	53	35	5	85	55	U	117	75	u
22	16	Synchronous idle	54	36	6	86	56	V	118	76	v
23	17	End trans. block	55	37	7	87	57	W	119	77	w
24	18	Cancel	56	38	8	88	58	X	120	78	x
25	19	End of medium	57	39	9	89	59	Y	121	79	y
26	1A	Substitution	58	3A	:	90	5A	Z	122	7A	z
27	1B	Escape	59	3B	;	91	5B	[123	7B	{
28	1C	File separator	60	3C	<	92	5C	\	124	7C	
29	1D	Group separator	61	3D	=	93	5D]	125	7D	}
30	1E	Record separator	62	3E	>	94	5E	^	126	7E	~
31	1F	Unit separator	63	3F	?	95	5F	_	127	7F	□

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Appendix B

The ECS17 Potato Prayers

- 1) Thou shalt not say “there exists k ” without mentioning the domain of k .
- 2) Thou shalt not say “it is obvious”
- 3) If p and q are two propositions, then $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$. This is the basis for the proof by contrapositive.
- 3) If p and q are two propositions, then $p \rightarrow q \Leftrightarrow \neg p \vee q$. This is the basis for the proof by contradiction.
- 4) An integer n is even if and only if there exists an integer k such that $n = 2k$. We say also that n is a multiple of 2.
- 5) An integer n is odd if and only if there exists an integer k such that $n = 2k + 1$.
- 6) BEWARE of divisions and square roots when you are working with integers.

Proofs that you can use without proving them again

We can use the following results without having to validate them:

- 1) Let n be an integer. Then:
 - a) If n is even, then $n + 1$ is odd
 - b) if n is odd, then $n + 1$ is even
- 2) Let n be an integer. Then:
 - a) n is even, if and only if n^2 is even
 - b) n is odd, if and only if n^2 is odd
- 3) $\forall n \in \mathbb{Z}$, $n(n + 1)$ is even.
- 4) $\sqrt{2}$ is irrational.