Review problems

ECS 17 (Winter 2025)

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1 Simple propositions

For each proposition on the left, indicate if it is a tautology or not:

Table 1: Propositional logic

Proposition	Tautology (Yes/ No)
$(\neg(p \wedge q)) \leftrightarrow (\neg p \vee \neg q)$	
$(\neg(p \wedge q)) \leftrightarrow (\neg p \wedge \neg q)$	
$(\neg(p \vee q)) \leftrightarrow (\neg p \wedge \neg q)$	
if $6^2 = 36$ then $2 = 3$	
if $6^2 = -1$ then $36 = -1$	

2 Knights and Knaves

A very special island is inhabited only by Knights and Knaves. Knights always tell the truth, while Knaves always lie. You meet three inhabitants: Alex, John and Sally. Alex says, "If John is a Knight then Sally is a Knight". John says, "Alex is a Knight and Sally is a Knave". Can you find what Alex, John, and Sally are? Explain your answer.

3 Proofs: direct, indirect, and contradictions

3.1 Different methods of proofs

Let n be an integer. Show that if $3n^2 + 2n + 9$ is odd, then n is even using a direct, indirect, and proof by contradiction.

3.2 Proof by contradiction

Let n be a strictly positive integer. Show that $\frac{2n+1}{2n+4}$ is not an integer

3.3 Proof by contradiction

Let n be a strictly positive integer. Show that if $\sqrt{n^2+1}$ is not an integer.

4 Proofs by induction

4.1 Identity

a) Show that $1+3+\ldots 2n-1=n^2$, for all $n \geq 1$.

b) Show that $\displaystyle\sum_{k=1}^{n} \frac{1}{4k^2-1} = \frac{n}{2n+1}$ for all integer $n \geq 1$.

4.2 Multiples

For the next two problems, we say that an integer n is a multiple of an integer m if and only if there exist an integer k such that $n = km$.

a) Show that $(7^n - 2^n)$ is a multiple of 5 for all integer $n \geq 1$.
b) Show that $n(2n+1)(7n+1)$ is a multiple of 6 or all integer $n \geq 1$.

4.3 Stamps: 1

Use induction to prove that any postage of n cents (with $n \geq 30$) can be formed using only 6–cent and 7–cent stamps.

4.4 Stamps: 2

Use induction to prove that any postage of n cents (with $n \geq 18$) can be formed using only 3–cent and 10–cent stamps.

4.5 Other

Prove by induction that for all $n \geq 1$, there exist two strictly positive integers a_n and b_n such that $(1+\sqrt{2})^n = a_n + b_n\sqrt{2}$.

4.6 Fibonacci

Let f_n be the Fibonacci numbers. show that $f_{n-1}f_{n+1} - f_n^2 = (-1)^n$, for all $n > 1$.