Data, Logic, and Computing

ECS 17 (Winter 2023)

Patrice Koehl
koehl@cs.ucdavis.edu

February 8, 2023

Midterm 1: solutions

Part 1 *(6 questions, each 5 points; total 30 points)*

1) *How many songs could you store on 1GByte, if those songs were each 5 min long, sampled at 44.1 kHz, with each data point stored on 16 bits, in stereo (i.e., with two microphones)? Assume no compression.*

   a) *About 10 songs*
   b) *About 20 songs*
   c) *About 40 songss*
   d) *About 60 songs*

   Let us look first at the space required to store one song:

$$S = 5[mins] \times 60 \frac{[s]}{[mins]} \times 44100 \frac{1}{[s]} \times 16[bits] \times 2[stereo]$$
$$S = 423360000 \quad [bits]$$
$$S = 52920000 \quad [bytes]$$

   As 1GB = 1073741824 bytes, we can store $\frac{1073741824}{52920000} \approx 20$ songs.

2) *2) Let X be the number with the hexadecimal representation #89 and Y the number whose binary representation is $(1111111)_2$; which of these numbers T (in hexadecimal form) satisfies $X - T = Y$?*

   a) *#A*
   b) *#B*
   c) *#C*
   d) *#D*

   Note that:

a) $X = \#89 = 8 * 16 + 9 = 137$ (decimal)
b) $Y = (1111111)_2 = 127$ (decimal)

Therefore $T = X - Y = 137 - 127 = 10$ (decimal) $= \#A$.

3) *The hexadecimal equivalent of* $(1110010)_2$ *is*

a) *#82*
b) *#71*
c) *#72*
d) *#F2*

Note that:

a) $(1110010)_2 = 0111 \quad 0010$
b) $0111 = \#7$ and $0010 = \#2$
c) Therefore $(1110010)_2 = \#72$.

4) *The heart rate of a hummingbird can go as high as 1260 beats per minute. Which of these sampling rates would be appropriate to monitor this heart rate? (Circle all that applies)*

a) *1 Hz*
b) *21 Hz*
c) *42 Hz*
d) *45 Hz*

1260 beats per minute = 21 beats per second = 21 Hz. Therefore we need a sampling rate strictly greater than $2 \times 21 = 42$ Hz; the correct answer is d).

5) *If we sum the ASCII representations of the letters in the word A_ (where _ is unfortunately unknown), we get the hexadecimal #94. What is this letter currently represented with _ ?*

a) *S*
b) *T*
c) *M*
d) *L*

The letter A is represented by the decimal 65. $\#94 = 9 \times 16 + 4 = 148$. Therefore the unknown letter _ is represented with the decimal $148 - 65 = 83$, which is the letter S.

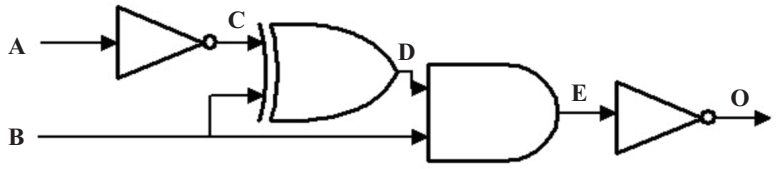
6) *You take a picture with a digital camera, and you know that this picture requires 64 Mbytes of storage (without compression). Assuming that each pixel is stored on 32 bits, how many pixels do the image contain?*

a) *12 Mega pixles*
b) *16 Mega pixels*
c) *2 Mega pixels*
d) *32 Mega pixels*

Each pixel is stored on 32 bits = 4 bytes. Therefore the image contains $\frac{64}{4} = 16$ Mega pixels.

Part II *(2 problems, each 10 points; total 20 points)*

1) *1) Complete the logic table corresponding to the logic gate shown below. Which gate is it equivalent to?* ***(10 points)***



A	B	$C = \bar{A}$	$D = C \oplus B$	$E = D \cdot B$	$O = \bar{E}$
1	1	0	1	1	0
1	0	0	0	0	1
0	1	1	0	0	1
0	0	1	1	0	1

This gate is equivalent to the NAND gate..

2) *Using only a logic table, show that* $\overline{\overline{A+B} \cdot (\overline{B}+C)} = A + B$ ***(10 points)***

Let $LHS = \overline{\overline{A+B} \cdot (\overline{B}+C)}$ and $RHS = A + B$. We build the logic table:

A	B	C	$A+B$	$\overline{A+B}$	$\overline{B}$	$\bar{B}+C$	$\overline{A+B} \cdot (\overline{B}+C)$	$LHS = \overline{\overline{A+B} \cdot (\overline{B}+C)}$	$RHS = A+B$
1	1	1	1	0	0	1	0	1	1
1	1	0	1	0	0	0	0	1	1
1	0	1	1	0	1	1	0	1	1
1	0	0	1	0	1	1	0	1	1
0	1	1	1	0	0	1	0	1	1
0	1	0	1	0	0	0	0	1	1
0	0	1	0	1	1	1	1	0	0
0	0	0	0	1	1	1	1	0	0

Therefore $LHS = RHS$.

Part III *(2 problems, each 10 points; total 20 points)*

a) *A very special island is inhabited by knights, knaves, and spies. Knights always tell the truth, knaves always lie, and spies may tell the truth or lie. This island is special as its empress has set up the following rule: "A knight can only marry a knave and a knave can only marry a knight". Clearly, a spy can only marry a spy. You meet two couples, let us call them Mr. and Mrs. Davis, and Mr and Mrs Dixon. Mr. Davis says, "Mr. Dixon is a knight", Mrs. Davis says, "This is true: Mr Dixon is a knight",and Mrs. Dixon says "I agree: my husband is a knight". What are each of the four people ? Show your work* ***(10 points)***

We build a truth table. Mr. and Mrs. Davis and Mr. and Mrs. Dixon can be Knights, Knaves, or spies, but do follow their empress' rule. This gives 9 possibilities for the couple.

Let $S_{DavisMr}$ be: "Mr Dixon is a knight".

Let $S_{DavisMrs}$ be: "Mr. Dixon is a knight"

Let $S_{DixonMrs}$ be: "Mr. Dixon is a knight"

Line #	Mr. Davis	Mrs. Davis	Mr Dixon	Mrs. Dixon	$S_{DavisMr}$	$S_{DavisMrs}$	$S_{DixonMrs}$
1	Knight	Knave	Knight	Knave	True	True	True
2	Knight	Knave	Knave	Knight	False	False	False
3	Knight	Knave	Spy	Spy	False	False	False
4	Knave	Knight	Knight	Knave	True	True	True
5	Knave	Knight	Knave	Knight	False	False	False
6	Knave	Knight	Spy	Spy	False	False	False
7	Spy	Spy	Knight	Knave	True	True	True
8	Spy	Spy	Knave	Knight	False	False	False
9	Spy	Spy	Spy	Spy	False	False	False

We can eliminate:

Line 1: Mrs Davis would be a knave that tells the truth

Line 2: Mr Davis would be a knight that lies

Line 3: Mr Davis would be a knight that lies

Line 4: Mr Davis would be a knave that tells the truth

Line 5: Mrs Davis would be a knight that lies

Line 6: Mrs Davis would be a knight that lies

Line 7: Mrs Dixon would be a knight that lies

Line 8: Mrs Dixon would be a knave that tells the truth

Therefore Mr Davis, Mrs Davis, Mr. Dixon, and Mrs. Dixon are all spies.

2) *You arrive in a country called Transylvania whose inhabitants are humans and vampires. Humans always tell the truth, while vampires always lie. However, both humans and vampires can be sane or insane. If an inhabitant is insane, she will believe that a truth statement is false, and a false statement is true. Sane inhabitants believe that truth statements are true and false statements are false. Thus sane humans and insane vampires make only true statements, while insane humans and sane vampires make only false statements. You meet two inhabitants, Alex and Bill. You know that one of them is a human, and the other is a vampire. Alex tells you: "we are both insane", while Bill tells you that "at least one of us is sane". From this, can you find which one is the vampire? (**10 points**)*

We build a truth table. Alex and Bill can each be a human or a vampire, sane or insane. This would give 16 possibilities for the pair. However, we know that one is human, and the other one is a vampire: this reduces the table to 8 possibilities.

Let S_{Alex} be: "We are both insane".

Let S_{Bill} be: "At least one of us is sane"

We can eliminate:

Line #	Alex	Bill	S_{Alex}	S_{Bill}
1	Human, Sane	Vampire, Sane	False	True
2	Human, Sane	Vampire, Insane	False	True
3	Human, Insane	Vampire, Sane	False	True
4	Human, Insane	Vampire, Insane	True	False
5	Vampire, Sane	Human, Sane	False	True
6	Vampire, Sane	Human, Insane	False	True
7	Vampire, Insane	Human, Sane	False	True
8	Vampire, Insane	Human, Insane	True	False

Line 1: Alex would be a sane human that lies

Line 2: Alex would be a sane human that lies

Line 3: Bill; would be a sane vampire that tells the truth

Line 4: Alex would be an insane human that tells the truth

Line 6: Bill would be an insane human that tells the truth

Line 7: Alex would be an insane vampire that lies

Both lines 5 and 8 are compatible with the premises. In both cases, Bill is a human and Alex is a vampire. We do not know however if they are sane or not.