Data, Logic, and Computing

ECS 17 (Winter 2026)

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February 5, 2026

Midterm 1: solutions

Part 1 *(6 questions, each 5 points; total 30 points)*

1) *1) How much space would you need to store a 5 min song that has been sampled at 44.1 kHz, with each data point stored on 24 bits, in stereo (i.e. with two microphones)? Assume no compression.*

   a) *About 80 Gbytes*

   b) *About 80 Mbytes*

   c) *About 106 Mbytes*

   d) *About 800 Kbytes*

Let us look at the space required to store one song:

$$S = 5[mins] \times 60\frac{[s]}{[mins]} \times 44100\frac{1}{[s]} \times 24[bits] \times 2[stereo]$$
$$S = 635,040,000 \quad [bits]$$
$$S = 79,380,000 \quad [bytes]$$
$$S \approx 80MBytes.$$

2) *2) Let X be the number with the hexadecimal representation #AA and Y the number whose binary representation is #9D; which of these numbers T (in hexadecimal form) satisfies X − T = Y?*

   a) *#A*

   b) *#B*

   c) *#C*

   d) *#D*

Note that:

a) $X = \#AA = 10 * 16 + 10 = 170$ (decimal)

b) $Y = \#9D = 9 * 16 + 13 = 157$ (decimal)

Therefore $T = X - Y = 170 - 157 = 13$ (decimal) $= \#D$.

3) *Which of these bytes represents the letter P (uppercase) based on the ASCII code?*

   a) *01010001*

   b) *10100001*

   c) *01010010*

   d) *10100010*

   e) *None of the above, as P is represented with 01010000*

This was a trick question! Note that P is represented with the hexadecimal $\#50$, meaning that the last 4 bits of its binary representation need to be 0000.

   a) $(010110001)_2 = \#51$

   b) $(10100001)_2 = \#A1$

   c) $(01010010)_2 = \#52$

   d) $(10100010)_2 = \#A2$

4) *4) A cooling fan in an industrial machine rotates at a maximum speed of 1200 revolutions per minute (rpm). You need to install a vibration sensor to monitor the fan's rotation frequency. Which of the following sampling rates is the most appropriate to monitor correctly the fan?*

   a) *20 Hz*

   b) *30 Hz*

   c) *40 Hz*

   d) *50 Hz*

1200 revolutions per minute $=$ 20 revolutions per second $=$ 20 Hz. Therefore we need a sampling rate strictly greater than $2 \times 20 = 40$ Hz; the correct answer is d).

5) *5) We have a word with 2 letters. If we sum the ASCII representations of these 2 letters, we get $\#96$. If, instead, the subtract the ASCII representation of the first letter from the ASCII representation of the second letter, we get $\#8$. What is the word?*

   a) *GO*

   b) *OG*

   c) *go*

   d) *og*

Let x and y be the ASCII representations of the two letters. We know that $x + y = \#96 = 150$ (decimal), and $y - x = 8$ (decimal). Therefore $x = 71$ and $y = 79$, end the two letters are G and O.

6) *6) Which binary number comes right after the binary number 101111?*
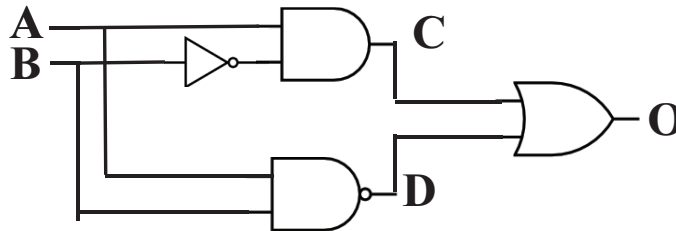
Part II *(2 problems, each 10 points; total 20 points)*

1) *1) Complete the logic table corresponding to the logic gate shown below. Convert it into a Boolean expression ? (**10 points**)*
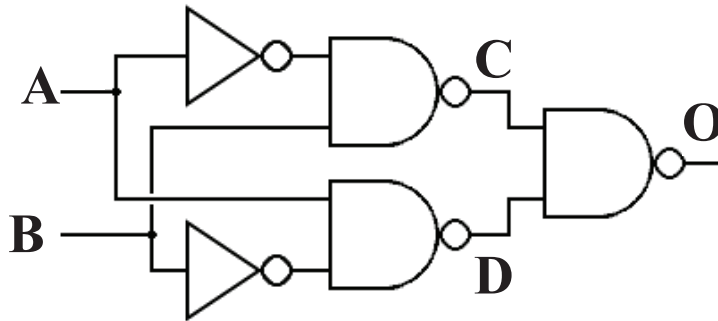
A	B	$C = \overline{\overline{A} \cdot B}$	$D = \overline{A \cdot \overline{B}}$	$O = \overline{C \cdot B}$
1	1	1	1	0
1	0	1	0	1
0	1	0	1	1
0	0	1	1	0

$O = \overline{C \cdot D} = \overline{C} + \overline{D} = \overline{\overline{\overline{A} \cdot B}} + \overline{\overline{A \cdot \overline{B}}} = \overline{A} \cdot B + A \cdot \overline{B}$. This gate is equivalent to the XOR gate.

2) *An engineer hands you a piece of paper with the following Boolean expression on it, and tells you to build a gate circuit to perform that function: $O = A \cdot \overline{B} + \overline{A} \cdot B$. Draw a logic gate circuit for this function. Represent its logic table. Can you find a simpler logic gate that would perform the same operation on A and B?*

This is the NAND gate.

A	B	$C = A \cdot \overline{B}$	$D = \overline{A} \cdot B$	$O = C + D$
1	1	0	0	0
1	0	1	1	1
0	1	0	1	1
0	0	0	1	1

Part III *(2 problems, each 10 points; total 20 points)*

1) *You encounter a problem on an exam with only answer choices:*

   a) *Option 1*

   b) *Option 1 or Option 2, or both*

   c) *Option 2 or Option 3, or both*

   You do not know what those options are, as the question has been omitted, but you know that only one answer (a, b, or c) is possible. Can you find that answer? Explain your reasoning.

   There are two was to think about this problem:

   a) : **The traditional way with a truth table**
   There are 3 options, and each option can be true or false. We build a truth table and check the validity of each answer:

Line	Option 1	Option 2	Option 3	Answer a	Answer b	Answer c
1	T	T	T	T	T	T
2	T	T	F	T	T	T
3	T	F	T	T	T	T
4	T	F	F	T	T	F
5	F	T	T	F	T	T
6	F	T	F	F	T	T
7	F	F	T	F	F	T
8	F	F	F	F	F	F

   As only one answer is correct, line 7 is the only correct one, and Option 3 is true and answer c is true.

   b) **A slightly faster way**

   i) If option 1 was correct, answers a) and b) would be valid: No, as only one answer is possible. Therefore option 1 is wrong.

   ii) If option 2 was correct, answers b) and c) would be valid: No, as only one answer is possible. Therefore option 2 is wrong.

   iii) Therefore, only option 3 is correct, and the answer is c).

2) *On twilight island, there are two types of residents: day-knights and knaves. Day-knights tell the truth during the day and lie at night, while knaves always lie. At a time when you don't know if it is day or night, you encounter Alice and Bob. Each makes a statement. Alice says "Bob is a day-knight" and Bob says "Alice is a knave". Is it day or night? What are Alice and Bob?.*

We build a truth table. Alex and Bill can each be a day-knight (DK) or a knave (N). It can be day (d), or night (n). This would give 8 possibilities.

Let S_1 be: "Bob is a day-knight".

Let S_2 be: "Alice is a knave"

Line #	Time	Alice	Bob	S_1	S_2
1	d	DK	DK	True	False
2	d	DK	N	False	False
3	d	N	DK	True	True
4	d	N	N	False	True
5	n	DK	DK	True	False
6	n	DK	N	False	False
7	n	N	DK	True	True
8	n	N	N	False	True

We can eliminate:

Line 1: Bob would be a day-knight that lies during the day

Line 2: Alice would be a day-knight that lies during the day

Line 3: Alice would be a knave that tells the truth

Line 4: Bob would be a knave that tells the truth

Line 5: Alice would be a day-knight that tells the truth during the night

Line 7: Alice would be a knave that tells the truth

Line 8: Bob would be a knave that tells the truth

Only line 6 is compatible with the premises. It is night, Alice is a day-knight, and Bob is a knave.