**ECS 17: Data, Logic, and Computing**

**Midterm**

**February 23, 2022**

***Notes:***

1. Midterm is open book, open notes…
2. You have 50 minutes, no more: I will strictly enforce this.
3. You can answer directly on these sheets (preferred), or on loose paper.
4. Please write your name at the top right of at least the first page that you turn in!
5. Please, check your work!

**Proofs**

**Exercise 1 (1 question, 10 points)**

Let *a* and *b* be two real numbers, with  and . Use a **proof by contradiction** to show that if , then .

**Exercise 2 (2 questions, each 10 points; total 20 points)**

**1)** Let *a* and *b* be two integers. Show that if $a+b\sqrt{2}=0$, then *a* =0 and *b*=0.

**2)** Let *m, n, p, q* be four integers. Show that $m+n\sqrt{2}=p+q\sqrt{2}$ if and only if *m=p* and *n=q*.

**Exercise 3 (1 question, 10 points)**

Let *x* be a real number. Show that $\left|x-1\right|\leq x^{2}-x+1$,

where $\left|\right|$ stands for the absolute value, defined as

$$\left|y\right|=\left\{\begin{array}{c}y if y\geq 0\\-y if y<0\end{array}\right.$$

for *y* real number.

**Exercise 4 (1 question, 10 points)**

Let *a* and *b* be two integers. Show that if $\left(a^{2}+b^{2}\right)^{2}$ is even, then *a+b* is even.

**Exercise 5 (1 question, 10 points)**

Let *a* and *b* be two real numbers. Show that if *a ≠ -1* AND *b ≠ -1* then $a+b+ab\ne -1$

(*Hint:* $a+b+ab=\left(a+1\right)\left(b+1\right)-1$).

**Exercise 6 (1 question, 10 points)**

Let *a* and *b* be two integers. Show that $a^{2}-4b\ne 2$

**Appendix**

***Theorems that you are allowed to use:***

a) Let *n* be an integer. *n2* is even if and only if *n* is even

b) Let *a* and *b* be two integers. If *a2 + b2* $ $is even, then $a+b$ is even

c) $∀n\in N,$ *n(n+1)* is even

d) $\sqrt{2}$ is irrational

e) Let *n* be an integer. *n* is even if and only if there exists an integer *k* such that *n = 2k*.

f) Let *n* be an integer. *n* is odd if and only if there exists an integer *k* such that *n = 2k+1*.

g) Let *n* be a positive integer. *n* is a perfect square if and only if there exists an integer *k* such that *n = k2*.