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ECS 17: Data, Logic, and Computing
Midterm 2
February 26, 2025

Notes:

1) Midterm is open book, open notes…
2) You have 50 minutes, no more: I will strictly enforce this.
3) The midterm is graded over 50 points.
4) You can answer directly on these sheets (preferred), or on loose paper.
5) Please write your name at the top right of at least the first page that you turn in!
6) Please, check your work!

Exercise 1 (10 points)

Let *a, b,* and *c* be 3 integers. Using the method of proof of your choice, show that if *abc* is even, then at least one of *a, b*, or *c* must be even.

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Exercise 2 (10 points)

Let *n* be an integer. Give an **indirect proof** of the proposition, if $n^2 + 8n$ is odd then $5n$ is odd.

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Exercise 3 (1 question, 10 points)

Let *n* be an integer. Use a **proof by contradiction** to show that $\frac{n^3+n^2+3}{2n^2+6}$ is not an integer.

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Exercise 4 (1 question, 10 points)

Let *n* be an integer. Use **a direct proof** to show that if *n* is odd, then there exists an integer *m* such that $n^2 = 8m + 1$.

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Exercise 5 (1 question, 10 points)

Let n be an integer. Show that $4n+3$ is not a perfect square. (*An integer a is a perfect square if and only if there exists an integer b such that $a=b^2$*).

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Appendix

The ECS17 Potato Prayers

1) Thou shalt not say "there exists k" without mentioning the domain of k.
2) Thou shalt not say "it is obvious"
3) If p and q are two propositions, then $p \to q \Leftrightarrow \neg q \to \neg p$. This is the basis for the proof by contrapositive.
3) If p and q are two propositions, then $p \to q \Leftrightarrow \neg p \vee q$. This is the basis for the proof by contradiction.
4) An integer n is even if and only if there exists and integer k such that $n = 2k$. We say also that n is a multiple of 2.
5) An integer n is odd if and only if there exists and integer k such that $n = 2k + 1$.
6) BEWARE of divisions and square roots when you are working with integers.

Proofs that you can use without proving them again

We can use the following results without having to validate them:

1) Let n be an integer. Then:
   a) If n is even, then $n + 1$ is odd
   b) if n is odd, then $n + 1$ is even
2) Let n be an integer. Then:
   a) n is even, if and only if n^2 is even
   b) n is odd, if and only if n^2 is odd
3) $\forall n \in \mathbb{Z}, \quad n(n + 1)$ is even.
4) $\sqrt{2}$ is irrational.