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ECS 17: Data, Logic, and Computing
Midterm
February 25, 2026

Notes:

1) Midterm is open book, open notes…
2) You have 50 minutes, no more: I will strictly enforce this.
3) The midterm is graded over 60 points.
4) You can answer directly on these sheets (preferred), or on loose paper.
5) Please write your name at the top right of at least the first page that you turn in!

Part I: logic (1 question, 10 points)

Using a truth table, establish for the proposition below if it is a tautology, a contradiction, or neither.

$$(p \longrightarrow q) \leftrightarrow (\neg q \longrightarrow \neg p)$$

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Part II: proofs (4 questions, each 10 points; total 40 points)

1) Prove that if $7n^2+4$ is even then n^2+2 is even, where n is a natural number.

2) Let n be a natural number. Use a **direct proof** to show that if $6n^2 + 3n + 11$ is odd, then n is even.

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3) Let *n* be a natural number. Use a **proof by contradiction** to show that $\sqrt{4n+2}$ is not an integer.

4) Let *a* and *b* be 2 integers. Use a **direct proof** to show that if $a^2 + b^2$ is even, then $a^2 - b^2$ is even.

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Part III: Knights and Knaves (1 question, 10 points)

A very special island is inhabited only by Knights and Knaves. Knights always tell the truth, while Knaves always lie. You meet three inhabitants: Alex, John and Sally. Alex says, `John is a Knight, if and only if Sally is a Knave'. John says, `If Sally is a Knight, then Alex is a Knight'. Can you find what Alex, John, and Sally are? Explain your answer.

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Appendix

The ECS17 Prayers

1) Thou shalt not say "there exists k" without mentioning the domain of k.
2) Thou shalt not say "it is obvious"
3) If p and q are two propositions, then $p \to q \Leftrightarrow \neg q \to \neg p$. This is the basis for the proof by contrapositive.
4) If p and q are two propositions, then $p \to q \Leftrightarrow \neg p \vee q$. This is the basis for the proof by contradiction.
5) An integer n is even if and only if there exists and integer k such that $n = 2k$. We say also that n is a multiple of 2.
6) An integer n is odd if and only if there exists and integer k such that $n = 2k + 1$.
7) BEWARE of divisions and square roots when you are working with integers.

Proofs that you can use without proving them again

We can use the following results without having to validate them:

1) Let n be an integer. Then:
   a) If n is even, then $n+1$ and $n-1$ are odd
   b) if n is odd, then $n+1$ and $n-1$ are even
2) Let n be an integer. Then:
   a) n is even, if and only if n^2 is even
   b) n is odd, if and only if n^2 is odd
3) $\forall n \in Z, n(n+1)$ is even.
4) $\sqrt{2}$ is irrational.

Identities

Let a and b be two real numbers:

1) $(a+b)^2 = a^2 + 2ab + b^2$
2) $(a-b)^2 = a^2 - 2ab + b^2$
3) $a^2 - b^2 = (a-b)(a+b)$
4) **Completing the square:** $a^2 + b^2 = (a+b)^2 - 2ab$