Data, Logic, and Computing

ECS 17 (Winter 2025)

Patrice Koehl
koehl@cs.ucdavis.edu

January 15, 2025

Homework 1

Exercise 1

Eighty four biscuits are to be fed to twelve pets; each pet is either a cat or a dog. Each dog is to get eight biscuits, and each cat is to get five. How many dogs and how many cats are there?

It is best to formalize the problem. Let C be the number of cats, and D the number of dogs. We translate the information we have as relationships on C and D:

- There are 12 pets total: $C + D = 12$
- Each dog eats 8 biscuits, each cat eats 5 biscuits, and there are 80 biscuits: $8D + 5C = 84$

The corresponding system of equation

$$D + C = 12$$
$$8D + 5C = 84$$

leads to $D = 8$ and $C = 4$.

Interestingly, there is another way to solve this problem. Each pet eats at least 5 biscuits. As there are 12 pets, this leads to 60 biscuits, and therefore there are only 24 biscuits left. Those biscuits are then given to dogs, three per dog... and therefore there are 8 dogs, and consequently 4 cats.

Exercise 2

A surfboard and a swimsuit cost \$2100 in total. The surfboard costs \$2,000 more than the swimsuit. How much does the surfboard cost?

Intuitively, we would say the surfboard costs \$2000.0 and the swimsuit costs \$100.00 but we would be wrong!. We need a more systematic approach to solve this problem. Let x be the cost of the surfboard, and y the cost of the swimsuit. We translate the two sentences given to us as:

$$\begin{aligned} x + y &= 2100 \\ x &= y + 2000 \end{aligned}$$

The solution to this system of equations is $x = \$2050$ and $y = \$50$.

Exercise 3

You are about to leave for holiday, but you forgot socks! You race back to your room, but the power is off so you can't see sock colors. Never mind, because you remember that in your drawer there are ten pairs of green socks, ten pairs of black socks, and eleven pairs of blue socks, but they are all mixed up.

What is the minimum number of socks you need to take before you can be sure to have at least one matching pair?

You only need to pick 4 socks: it is the colors that matter, not the number of socks in each color.

More "rigorously". We check systematically when we are guaranteed to have a pair of matching socks. Obviously, we need at least 2 socks.

2 socks: While we could get lucky, we cannot guarantee that we will have a pair of matching socks as those two socks could be of different colors (there are 3 possible colors).

3 socks: If we need a third sock, this means that the first 2 socks are of different colors. When we pick the 3rd sock, we could be lucky with this sock to be the same color of one of the first two socks. However, it is possible that this third sock is of the 3rd color possible. Hence we cannot guarantee that 3 socks are enough.

4 socks: If we need 4 socks, this means that the first 3 socks are of different colors. Now the 4th socks has to be of one of those 3 colors, as there are only 3 colors available. Then we can guarantee that if we pick 4 socks, we will have a pair of matching socks.

If we pick more than 4 socks, we only need to look at 4 of them to find a matching pair. As the question was to identify the minimum number, this number is 4.

Exercise 4

You are on an island inhabited by three types of people: knights (always make true statements), knaves (always make false statements) and spies (sometimes make true statements and sometimes make false statements). You come across 3 people Adam, Ben and Carl. You know that one is a knight, one is a knave and one is a spy. They say the following:

- *Adam: "Carl is a knave"*
- *Ben: "Adam is a knight"*
- *Carl: "I am a spy"*

Determine which person is what or whether you do not have enough information.

The easiest approach to solve this problem is to use a table: T means that the person said the truth, and F that the person said a lie. Note that when we fill up the table, we need to consider that we have one Knight, one Knave, and one Spy.

Now we note that:

- line 1 cannot be correct: Adam would be a knight that lies.
- line 3 cannot be correct: Ben would be a knight that lies.

Line	Adam	Ben	Carl	Adam says	Ben says	Carl says
1	Knight	Knave	Spy	F	T	T
2	Knight	Spy	Knave	T	T	F
3	Knave	Knight	Spy	F	F	T
4	Knave	Spy	Knight	F	F	F
5	Spy	Knight	Knave	T	F	F
6	Spy	Knave	Knight	F	F	F

- line 4 cannot be correct: Carl would be a knight that lies.
- line 5 cannot be correct: Ben would be a knight that lies.
- line 6 cannot be correct: Carl would be a knight that lies.

Line 2 is the only possibility, i.e. Adam is a Knight, Ben is a Spy, and Carl is a Knave.

Exercise 5

You are on an island inhabited by two types of people: knights (always make true statements), and knaves (always make false statements). You come across 3 people Adam, Ben and Carl. They say the following:

- *Adam: "We are all three knaves"*
- *Ben: "Two of us are knaves and one of us is a knight"*
- *Carl: "I am a knight and the others are knaves"*

Determine which person is what or whether you do not have enough information.

The easiest approach to solve this problem is again to use a table: T means that the person said the truth, and F that the person said a lie. Note that when we fill up the table, we need to consider eight cases.

Line	Adam	Ben	Carl	Adam says	Ben says	Carl says
1	Knight	Knight	Knight	F	F	F
2	Knight	Knight	Knave	F	F	F
3	Knight	Knave	Knight	F	F	F
4	Knight	Knave	Knave	F	T	F
5	Knave	Knight	Knight	F	F	F
6	Knave	Knight	Knave	F	T	F
7	Knave	Knave	Knight	F	T	T
8	Knave	Knave	Knave	T	F	F

Now we note that:

- line 1-4 cannot be correct: Adam would be a knight that lies.
- line 5 cannot be correct: Ben would be a knight that lies.
- line 7 cannot be correct: Ben would be a knave that tells the truth.
- line 8 cannot be correct:Adam would be a knave that tells the truth.

Line 6 is the only possibility, i.e. Adam is a Knave, Ben is a Knight, and Carl is a Knave.