# Data, Logic, and Computing

ECS 17 (Winter 2025)

Patrice Koehl koehl@cs.ucdavis.edu

January 22, 2025

## Homework 3

### Exercise 1

We are on the island of knights and knaves. You meet three residents, Alex, Bill, and Claudia who make the following statements:

Alex says: "Bill is a knave and Claudia is a knight" Bill says: "Claudia is a knight and Alex is a knight" Claudia says: "I like cookies"

#### Does Claudia really like cookies?

We check all possible "values" for Alex, Bill, and Claudia, as well as the veracity of their statements.

Line number	Alex	Bill	Claudia	Alex says	Bill says
1	Knight	Knight	Knight	$\mathbf{F}$	Т
2	Knight	Knight	Knave	$\mathbf{F}$	$\mathbf{F}$
3	Knight	Knave	Knight	Т	Т
4	Knight	Knave	Knave	$\mathbf{F}$	$\mathbf{F}$
5	Knave	Knight	Knight	$\mathbf{F}$	$\mathbf{F}$
6	Knave	Knight	Knave	$\mathbf{F}$	$\mathbf{F}$
7	Knave	Knave	Knight	Т	$\mathbf{F}$
8	Knave	Knave	Knave	$\mathbf{F}$	F

We can eliminate:

- Line 1, as Alex would be a knight but he lies
- Line 2, as Alex would be a knight but he lies
- Line 3 as Bill would be a knight but he lies

- Line 4 as Alex would be a knight but he lies
- Line 5 as Bill would be a knight but he lies
- Line 6 as Bill would be a knight but he lies
- Line 7 as Alex would be a knave but he says the true

Therefore all three are knaves. Claudia then lies: she does not like cookies!

#### Exercise 2

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You meet three inhabitants: Alex, John and Sally. Alex says, "At least one of the following is true: that Sally is a knave or that I am a knight." John says, "Alex could claim that I am a knave." Sally claims, "Neither Alex nor John are knights." Can you find who is a knight and who is a knave?

We check all possible "values" for Alex, John and Sally, as well as the veracity of their statements.

Line number	Alex	John	Sally	Alex says	John says	Sally says
1	Knight	Knight	Knight	Т	$\mathbf{F}$	$\mathbf{F}$
2	Knight	Knight	Knave	Т	$\mathbf{F}$	$\mathbf{F}$
3	Knight	Knave	Knight	Т	Т	$\mathbf{F}$
4	Knight	Knave	Knave	Т	Т	$\mathbf{F}$
5	Knave	Knight	Knight	$\mathbf{F}$	Т	$\mathbf{F}$
6	Knave	Knight	Knave	Т	Т	$\mathbf{F}$
7	Knave	Knave	Knight	$\mathbf{F}$	$\mathbf{F}$	Т
8	Knave	Knave	Knave	Т	$\mathbf{F}$	Т

We can eliminate:

- Line 1, as John would be a knight but he lies
- Line 2, as John would be a knight but he lies
- Line 3 as John would be a knave but he says the true
- Line 4 as John would be a knave but he says the true
- Line 5 as Sally would be a knight but she lies
- Line 6 as Alex would be a knave but he says the true
- Line 8 as Alex would be a knave but he says the true

Line 7 is valid, and it is the only one. Therefore, both Alex and John are knaves, while Sally is a knight.

## Exercise 3

Find the output/ truth table for this logic gate circuit. Convert it into a Boolean expression



A	B	C	AB	B+C	AB(B+C)	Y
1	1	1	1	1	1	0
1	1	0	1	1	1	0
1	0	1	0	1	0	1
1	0	0	0	0	0	1
0	1	1	0	1	0	1
0	1	0	0	1	0	1
0	0	1	0	1	0	1
0	0	0	0	0	0	1

The corresponding Boolean expression is  $\overline{AB(B+C)}$ .

## Exercise 4

Find the output/ truth table for this logic gate circuit. Can you find a simpler logic gate that would perform the same operation on A and B?



A	B	$\overline{A}$	$\overline{AB}$	$\overline{A} + \overline{AB}$	$\overline{\overline{A}} + \overline{AB}$
1	1	0	0	0	1
1	0	0	1	1	0
0	1	1	1	1	0
0	0	1	1	1	0

The corresponding Boolean expression  $is\overline{A} + \overline{AB}$ . Note however that the output of this logic gate is exactly the ouput of the AND logic gate.

### Exercise 5

Let us play a logical game. You find yourself in front of three rooms whose doors are closed. Behind each door, there could be a Lady or a Tiger. There is only one Lady and two Tigers. There is one sign on each door; you are told that the sign on the door of the room containing the Lady is true, and that at least one of the other two signs is false Here are the signs:

Room I	Room II	Room III
A Tiger is in room II	A Tiger is in this room	A Tiger is in room I

Can you find what is inside each room?

Let us build the table for the possible options for the three doors. We then check the validity of the two statements on the door, and finally check the consistency of the truth values for those statements with what we know about their validity.

Line	Room I	Room II	Room III	Sign I	Sign II	Sign III	Compatibility
1	T - J	<b>T</b> :	<b>T</b> :	т	T	Б	V
1	Lady Tiger	1 iger Ladv	1 iger Tiger	т Е	т Е	г Т	res
$\frac{2}{3}$	Tiger	Tiger	Lady	T	T	T	No

Justification:

- Line 2 : There is a Lady in room II; the sign on that room should then be true, but it is false: incompatibility
- Line 3 : There are in room I and room II; one of the signs on these rooms should then be false, but they are both true: incompatibility

Therefore only line 1 is compatible: there is a Tiger in room II and room III and a Lady in room I!

#### Exercise 6

Which answer in the list below is the correct answer to this question?

- 1) All of the below
- 2) None of the below
- 3) All of the above

- 4) One of the above
- 5) None of the above
- 6) None of the above

We check the validity of each answer:

- Answer 1) If answer 1 is true, then both answer 4 and 5 would be true... but those two answers contradict each other. Therefore answer 1 is false.
- Answer 2) If answer 2 is true, then answer 5 is false, which would mean that all answers from 1 to 4 are true... but we know that answer is false. Therefore answer 2 is false.
- Answer 3) We know that answers 1 and 2 are false; therefore answer 3 cannot be true; it is false
- Answer 4) As answers 1, 2, and 3 are false, answer 4 is false.
- Answer 5) As answer 1, 2, 3, and 4 are false, answer 5 is true
- Answer 6) As answer 5 is true, answer 6 cannot be true.

Therefore, there is a single solution, Answer 5.