Homework 5: Solutions ECS 17 (Winter 2025)

February 4, 2025

Exercise 1

Construct a truth table for each of these compound propositions:

a) $A = (p \vee q) \rightarrow (p \oplus q)$

p	q	$p \vee q$	$p \oplus q$	A
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

b) $A = (p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$

p	q	r	$p \leftrightarrow q$	$\neg p$	$\neg r$	$\neg p \leftrightarrow \neg r$	A
T	T	T	T	F	F	T	F
T	T	F	T	F	T	F	T
T	F	T	F	F	F	T	T
T	F	F	F	F	T	F	F
F	T	T	F	T	F	F	F
F	T	F	F	T	T	T	T
F	F	T	T	T	F	F	T
F	F	F	T	T	T	T	F

c) $A = ((p \oplus q) \rightarrow (p \oplus \neg q))$

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	A
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Exercise 2

Construct a truth table for each of these compound propositions:

a) $A = (\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

p	q	r	$\neg p$	$\neg q$	$\neg p \leftrightarrow \neg q$	$q \leftrightarrow r$	A
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

b) $(p \oplus q) \wedge (p \oplus \neg q)$

p	q	$\neg q$	$p \oplus q$	$(p \oplus \neg q)$	$(p \oplus q) \wedge (p \oplus \neg q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F

Exercise 3

Show that the following is a tautology: $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$.

Let us define $A = (p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$. The proposition is $A \rightarrow r$.

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	A	$A \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	F	T	T	F	T
F	F	F	F	T	T	F	T

As $A \rightarrow r$ is always true, it is a tautology.

Exercise 4

Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are equivalent.

We show that these two statements have the same truth values:

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$	$p \leftrightarrow q$
T	T	T	F	F	F	T	T
T	F	F	F	T	F	F	F
F	T	F	T	F	F	F	F
F	F	F	T	T	T	T	T

As $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ always have the same truth values, they are equivalent.

Exercise 5

a) *Use a truth table to show that $(p \rightarrow q) \wedge (p \rightarrow r) \Leftrightarrow p \rightarrow (q \wedge r)$*

We show that the two statements $A = (p \rightarrow q) \wedge (p \rightarrow r)$ and $B = p \rightarrow (q \wedge r)$ have the same truth values:

p	q	r	$p \rightarrow q$	$p \rightarrow r$	A	$q \wedge r$	B
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	F	F	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

As the two statements A and B always have the same truth values, they are logically equivalent.

b) *Use logical equivalences to show the same thing, i.e. that $(p \rightarrow q) \wedge (p \rightarrow r) \Leftrightarrow p \rightarrow (q \wedge r)$*

We use logical equivalence.

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\begin{array}{rcl|c}
A & \Leftrightarrow & (p \rightarrow q) \wedge (p \rightarrow r) & \text{Problem definition} \\
 & \Leftrightarrow & (\neg p \vee q) \wedge (\neg p \vee r) & \text{property of implication} \\
 & \Leftrightarrow & \neg p \vee (q \wedge r) & \text{Distributivity} \\
 & \Leftrightarrow & p \rightarrow (q \wedge r) & \text{property of implication} \\
 & \Leftrightarrow & B &
\end{array}
$$

Exercise 6

The Fair Maiden Rowena wishes to wed. And her father, the Evil King Berman, has devised a way to drive off suitors. He has a little quiz for them, and here it is. It's very simple:

Three boxes sit on a table. The first is made of gold, the second is made of silver, and the third is made of lead. Inside one of these boxes is a picture of the fair Rowena. It is the job of the White Knight to figure out, without opening them, which one has her picture.

Now, to assist him in this endeavor there is an inscription on each of the boxes. The gold box says, "Rowena's picture is in this box." The silver box says, "The picture is not in this box." The lead box says, "The picture is not in the gold box." Only one of the statements is true. Which box holds the picture?

The simplest approach to solve this problem is to check systematically if the Gold box, the Silver box, or the Lead box contains the picture. In each case, we test the validities of the three statements.

Box with picture	Golden Box picture is in this box	Silver Box picture is not in this box	Lead Box picture is not in Gold box
Gold	True	True	False
Silver	False	False	True
Lead	False	True	True

If the prize were in the Gold or Lead box, two of the propositions would be true, whereas if the prize is in Silver Box, only one proposition would be true. The latter is therefore true, and the prize is in the Silver Box.