

Data, Logic, and Computing

ECS 17 (Winter 2025)

Patrice Koehl
koehl@cs.ucdavis.edu

February 10, 2025

Homework 6

Exercise 1

Let n be a positive integer. Show that n is even if and only if $7n + 4$ is even.

Exercise 2

Let a and b be two positive integers. Prove that if $n = ab$, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$

Exercise 3

Let m and n be two integers. Show that if $m > 0$ and $n \leq -2$, then $m^2 + mn + n^2 \geq 0$

Exercise 4

Let a and b be two integers. Show that if $a^2 + b^2$ is even, then $a + b$ is even:

- a) Using an indirect proof (proof by contrapositive)
- b) Using a proof by contradiction
- c) Using a direct proof

Exercise 5

This exercise relates to the inhabitants of the island of knights and knaves, where knights always tell the truth and knaves always lie. John and Bill are residents. John tells you: “if Bill is a knave, then I am a knight”, while Bill tells you: “if John is a knave and I am a knight, then $2+2=5$ ”. Can you say what John and Bill are?