

Data, Logic, and Computing

ECS 17 (Winter 2025)

Patrice Koehl
koehl@cs.ucdavis.edu

February 18, 2025

Homework 7

Exercise 1

Determine the truth values of the following statements; justify your answers:

- a) $\forall n \in \mathbb{N}, (n + 2) > n$
- b) $\exists n \in \mathbb{N}, 2n = 3n$
- c) $\forall n \in \mathbb{Z}, 3n \leq 4n$
- d) $\exists x \in \mathbb{R}, x^4 < x^2$

Exercise 2

Show that the following statements are true.

- a) Let x be a real number. Prove that if x^3 is irrational, then x is irrational.
- b) Let x be a positive real number. Prove that if x is irrational, then \sqrt{x} is irrational.
- c) Prove or disprove that if a and b are two rational numbers, then a^b is also a rational number.
- d) let n be a natural number. Show that n is even if and only if $3n + 8$ is even.
- e) Prove that either $4 \times 10^{769} + 22$ or $4 \times 10^{769} + 23$ is not a perfect square. Is your proof constructive, or non-constructive?

Note: for question e), a natural number n is a perfect square if there exists a natural number q such that $n = q^2$. For example, 4, 9, 16, 25, are all perfect squares while 2, 3, 5, 6,.... are not.

Exercise 3

Let n be a natural number and let a_1, a_2, \dots, a_n be a set of n real numbers. Prove that at least one of these numbers is greater than, or equal to the average of these numbers. What kind of proof did you use?

Exercise 4

Use Exercise 3 to show that if the first 10 strictly positive integers are placed around a circle, in any order, then there exist three integers in consecutive locations around the circle that have a sum greater than or equal to 17.