# Data, Logic, and Computing

ECS 17 (Winter 2025)

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### Homework 8

#### Exercise 1

Using induction, show that  $\forall n \in \mathbb{N}, \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$ 

## Exercise 2

Using induction, show that 
$$\forall n \in \mathbb{N}$$
,  $\sum_{i=1}^{n} i(i+1)(i+2) = \frac{n(n+1)(n+2)(n+3)}{4}$ .

## Exercise 3

Show that 
$$\forall n \in \mathbb{N}, n > 1, \sum_{i=1}^{n} \frac{1}{i^2} < 2 - \frac{1}{n}$$

## Exercise 4

Use a proof by induction to show that  $\forall n \in \mathbb{N}, n > 3, n^2 - 7n + 12 \ge 0$ .

#### Exercise 5

A sequence  $a_0, a_1, \ldots, a_n$  of natural numbers is defined by  $a_0 = 2$  and  $a_{n+1} = (a_n)^2$ ,  $\forall n \in \mathbb{N}$ . Find a closed form formula for the term  $a_n$  and prove that your formula is correct.

#### Exercise 6

Use the method of proof by induction to show that any amount of postage of 24 cents or more can be formed using just 5-cent and 7-cent stamps.