

# Good practices

1/8/25

①

## Exercise 1

Solve  $x^2 = 2$

if $x$ is integer,	no solution
$x$ rational number,	no solution,
$x$ positive real number	$\sqrt{2}$
$x$ real number	$\{\sqrt{2}, -\sqrt{2}\}$

## Exercise 2:

Let  $x$  be a real number.

Show that  $(x+5)^2 - (x+3)^2 = 4(x+4)$

Solution:

~~$$\begin{aligned} & |(x+5)^2 - (x+3)^2 = 4(x+4)| \\ & x^2 + 10x + 25 - x^2 - 6x - 9 = 4x + 16 \\ & 4x + 16 = 4x + 16 \\ & 16 = 16 \end{aligned}$$~~

I need to show ②

$$\underbrace{(x+5)^2 - (x+3)^2}_{LHS} = \underbrace{4(x+4)}_{RHS}$$

$$\begin{aligned} LHS &= (x+5)^2 - (x+3)^2 \\ &= x^2 + 10x + 25 - x^2 - 6x - 9 \\ &= 4x + 16 \end{aligned}$$

$$\begin{aligned} RHS &= 4(x+4) \\ &= 4x + 16 \end{aligned}$$

---

Therefore,  $LHS = RHS$

### Exercise 3

Let  $n$  be a natural number.

Show that  $n(n+1) > n^2$

$$\text{Let } LHS = n(n+1)$$

$$RHS = n^2$$

Let us compute  $LHS - RHS$ :

$$\begin{aligned} LHS - RHS &= n(n+1) - n^2 \\ &= n^2 + n - n^2 \end{aligned}$$

$$= n > 0$$

Therefore,  $LHS > RHS$

## Exercise 4

(3)

A bottle of wine costs \$11.00

The wine itself costs \$10.00 more than the bottle. How much does the wine cost?

Let  $W$  be the cost of wine.

Let  $B$  be the cost of the bottle.

$$W + B = 11 \quad \textcircled{1}$$

$$W = 10 + B \quad \textcircled{2}$$

Solving:

Replacing  $\textcircled{2}$  into  $\textcircled{1}$ :

$$10 + B + B = 11$$

$$2B = 1 \rightarrow B = \$0.5$$

$$W = 10 + B = \$10.5$$

Exercise 5 Island of knights and knaves. (4)

On this island there are 2 types of inhabitants:

Knights that always tell the truth

Knaves that always lie.

You meet Bill and Sophie.

Bill: "We are both of the same type"

Sophie: "We are different".

Bill	Sophie	Bill says	Sophie says
Knight	<del>Knight</del>	True	<del>False</del> X
<del>Knight</del>	Knave	<del>False</del>	True X
Knave	Knight	False	True
<del>Knave</del>	Knave	<del>True</del>	False X

# Exercise 6:

We are still on the island of knights and knaves. We meet again two persons, Karl and Celia.

Karl: "We are both knaves"

Celia: "I like chocolate!"

Karl	Celia	Karl says
<u>Knight</u>	Knight	<u>False</u> X
<u>Knight</u>	Knave	<u>False</u> X
Knave	Knight	False
<u>Knave</u>	Knave	<u>True</u> X