Good practices

1/5/26 ①

Exercise 1

Let x be a real number. Show that $(x+1)^2 - x^2 = 2x+1$

What not to do:

$(x+1)^2 - x^2 = 2x+1$

$x^2 + 2x + 1 - x^2 = 2x+1$

$2x + 1 = 2x + 1$

$1 = 1$

Instead:

$LHS = (x+1)^2 - x^2 = 2x+1$

$RHS = 2x+1$

$LHS - RHS = 2x+1-(2x+1) = 0$

Therefore: $LHS = RHS$. ✓

Exercise 2:

Let n be a natural number. Show that $n(n+1) > n^2$.

Solution: Let $LHS = n(n+1) = n^2 + n$

$RHS = n^2$

$LHS - RHS = n^2 + n - n^2 = n > 0$

Therefore $LHS > RHS$

$\Leftrightarrow n(n+1) > n^2$ ✓

Exercise 3

A bottle of wine costs $11.00. The wine costs $10.00 more than the bottle. How much does the bottle costs?

Solution

W: cost of the wine

B: cost of the bottle.

$$\begin{cases} W + B = 11 & (1) \\ W = B + 10 & (2) \end{cases}$$

Replacing (2) in (1),

$$B + 10 + B = 11$$

$$2B = 1$$

$$\boxed{B = \$0.5}$$

Exercise 4

We are on the island of knights and knaves. There are 2 types of inhabitants, knights that always tell the truth, knaves that always lie. You meet Bill and Sophie:

Bill: "We are of the same type."

Sophie: "We are different"

Bill	Sophie	Bill says	Sophie says	
Knight	(Knight)	True	(False)	X
(Knight)	Knave	(False)	True	X
Knave	Knight	False	True	OK
(Knave)	Knave	(True)	False	X

Exercise 5

We are still on the island of Knights and Knaves. We meet again Bill and Sophie:

Bill: "We are both Knaves"

Sophie: "I like chocolate"

Does Sophie like chocolate?

Bill	Sophie	Bill says	
Knight	Knight	False	×
Knight	Knave	False	×
Knave	Knight	False	
Knave	Knave	True	×

Sophie likes chocolate.

Exercise 6

In a game show, you are presented with 3 boxes

A: The prize is here

B: The prize is not in A

C: The prize is not here

Only one statement is true.

Prize	Box A	Box B	Box C
A	True	False	True
B	False	True	True
C	False	True	False