

Exercise 1:

Let x be a real number.

Show that $(x+1)^2 - (x-1)^2 = 4x$

Solution:

~~$(x+1)^2 - (x-1)^2 = 4x$~~

~~$x^2 + 2x + 1 - (x^2 - 2x + 1) = 4x$~~

~~$x^2 + 2x + 1 - x^2 + 2x - 1 = 4x$~~

~~$4x = 4x$~~

~~$x = x$ true.~~

Let $LHS = (x+1)^2 - (x-1)^2$

$RHS = 4x$

$$\begin{aligned} LHS &= (x+1)^2 - (x-1)^2 = (x^2 + 2x + 1) - (x^2 - 2x + 1) \\ &= x^2 + 2x + 1 - x^2 + 2x - 1 \\ &= 4x \end{aligned}$$

$$LHS - RHS = 4x - 4x = 0$$

$$LHS = RHS$$

Exercise 2

(2)

Let n be a natural number.

Show that $n(n+1) \geq n^2$

Solution:

n is a natural number: $n > 0$

$$\text{Let } LHS = n(n+1) = n^2 + n$$

$$RHS = n^2$$

$$LHS - RHS = n^2 + n - n^2 = n > 0$$

$$LHS - RHS > 0$$

$$LHS - \cancel{RHS} + \cancel{RHS} > RHS$$

$$LHS > RHS$$

Exercise 3

A baseball bat and a ball cost \$11.00

The bat costs \$10.00 more than the ball.

How much does the ball cost?

let B : cost of the bat
 b : cost of the ball

$$B + b = 11 \quad (1)$$

$$B = b + 10 \quad (2)$$

Replace (2) into (1):

$$b + 10 + b = 11$$

$$2b = 1$$

$$b = \frac{1}{2} = 0.5$$

Exercise 4 You arrive on the island of Knights and Knaves. You meet 2 persons, Bill, and Sophie.

Bill: "We are of the same type"

Sophie: "We are different"

Bill	Sophie	S_A	S_S	
Knight	Knight	True	False	X
Knight	Knave	False	True	X
Knave	Knight	False	True	OK
Knave	Knave	True	False	X

Exercise 5: You are still on the island ⁽⁴⁾
of knights and knaves. You meet
John and Elsa.

John: "We are both knaves": S,

Elsa: "I like chocolate".

John	Elsa	S,	
<u>Knight</u>	Knight	<u>False</u>	X
<u>Knight</u>	Knave	<u>False</u>	X
Knave	Knight	False	
<u>Knave</u>	Knave	<u>True</u>	X