Discussion 2 1/14/26

I) Unsigned / signed binary numbers.

In our world,

101 next 102, previous 100

In the world of the computer.

101 next 110, previous 100

1000 next 1001 previous 0111

If you have an unsigned integer on N bits:

Largest: $2^N - 1$

How many different numbers: 2^N

N bit, but signed integers:

Largest:

N bits

Sign N-1 bits for the numbers.

Largest signed integer on N bits: $2^{N-1} - 1$

How many signed integers can you represent on N bits:

sign: N-1

without the sign, 2^{N-1}

+ numbers: 2^{N-1}

− numbers: 2^{N-1}

total: $2 \times 2^{N-1} = 2^N$

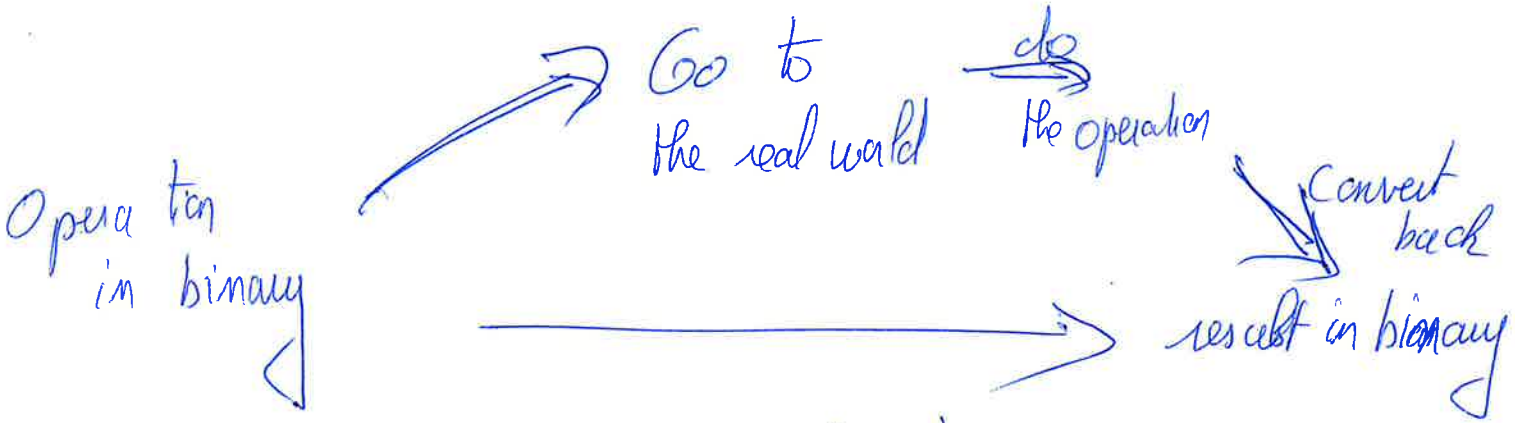
However, the number 0 has been counted twice:

answer: $\boxed{2^N - 1}$

Exercise 2

Let $A = (1010)_2$ and $B = (11011)_2$ (both unsigned integers).

Can you find the binary representation (unsigned) of the number X such that $A + X = B$



$A = (1010)_2 = (10)_{10}$

$B = (11011)_2 = (27)_{10}$

$X = B - A = 27 - 10 = 17$

$X = (10001)_2$

Exercise 3

Let $A = \#89$ and $B = (1111111)_2$

Find the hexadecimal representation of the number X that satisfies

$$X + B = A.$$

$$A = \#89 = 8 \times 16 + 9 = 137$$

$$D = (89)_{10} = 8 \times 10 + 9$$

$$B = (1111111)_2 = 2^7 - 1 = 127$$

$$X = A - B = 137 - 127 = 10$$

$$\boxed{X = \#A}.$$

Exercise 4

We are on the island of knights and knaves. We meet Sue and Bob.

Sue: "Bob is a knave"

Bob: "We are both knights".

K: knight and N: knave.

(1) Options (2) what is said. (3) Consistency.

Sue	Bob	S_1	S_2	
(K)	K	(F)	T	X
K	N	T	F	OK
N	(K)	F	(F)	X
(N)	N	(T)	F	X

Conclusion: Sue is a knight and Bob is a knave

Exercise 5

We are on the island of knights and knaves. We meet Sue, Bob and Mel.

Sue: "Only a knave would say that Bob is a knave" → Bob has to be a knight.

Bob: "It is false that Mel is a knave" → Mel is a knight.

Mel: "Bob would tell that I am a knave".

K: knight N: knave

① Options ② What is said ③ check

Sue	Bob	Mel	S_1	S_2	S_3	
K	K	K	T	T	F	X
K	K	N	T	F	T	X
K	N	K	F	T	T	X
K	N	N	F	F	F	X
N	K	K	T	T	F	X
N	K	N	T	F	T	X
N	N	K	F	T	T	X
N	N	N	F	F	F	OK

All 3 are knaves