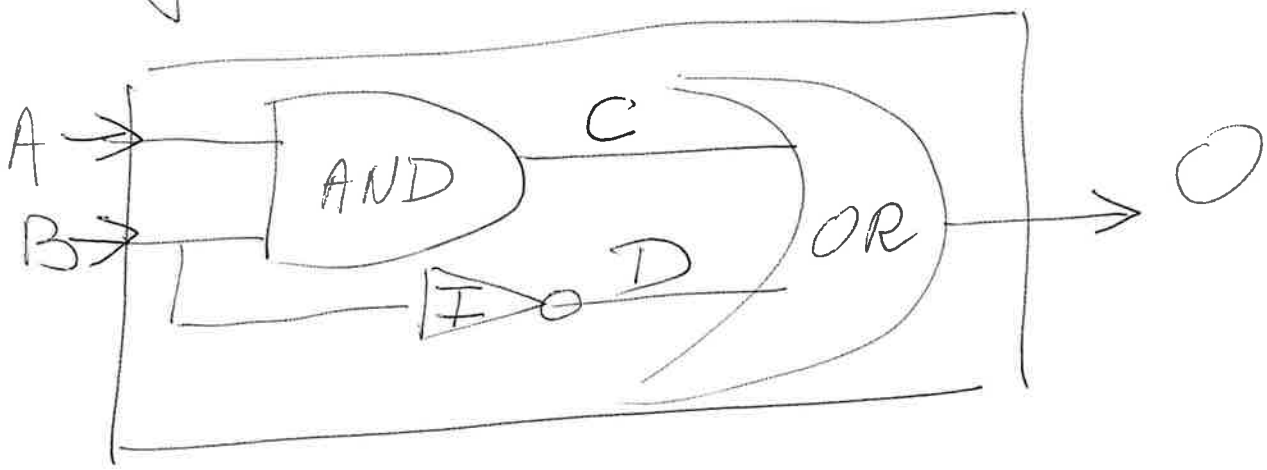
Exercise 1

Characterize the circuit:



Logic table:

A	B	C	D	O
1	1	1	0	1
1	0	0	1	1
0	1	0	0	0
0	0	0	1	1

$$O = C + D$$

$$= A \cdot B + \overline{B}$$

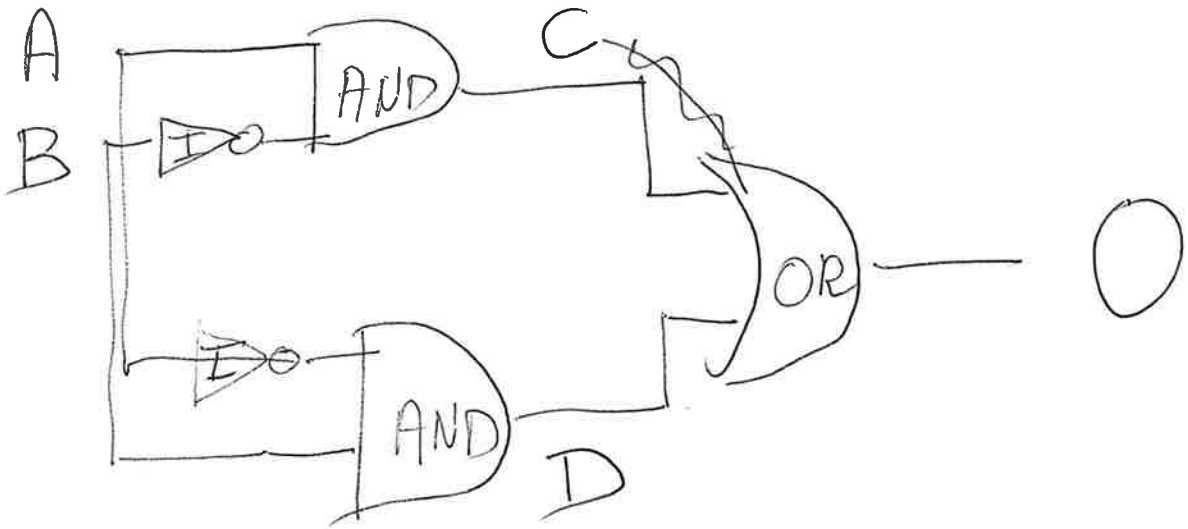
Exercise 2:

Build the diagram and logic table for the Boolean expression

$$O = A \cdot \overline{B} + \overline{A} \cdot B$$

Can you simplify it?

Step 1: build the circuit.



A	B	$\overline{A}$	$\overline{B}$	C	D	O
1	1	0	0	0	0	0
1	0	0	1	1	0	1
0	1	1	0	0	1	1
0	0	1	1	0	0	0

Therefore:

$$A \oplus B = A\overline{B} + \overline{A}B$$

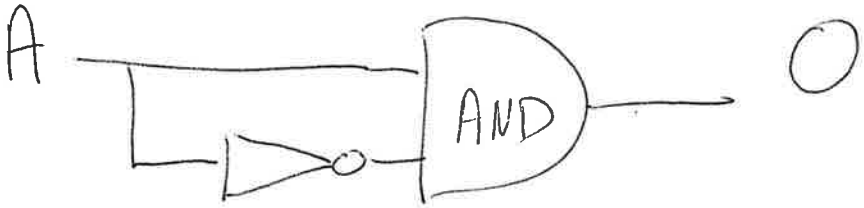
$$\overline{A \oplus B} = \overline{A\overline{B} + \overline{A}B}$$

$$= \overline{A\overline{B}} \cdot \overline{\overline{A}B}$$

$$= \left(\overline{A} + \overline{\overline{B}}\right) \cdot \left(\overline{\overline{A}} + \overline{B}\right)$$

$$= \left(\overline{A} + B\right) \cdot \left(A + \overline{B}\right)$$

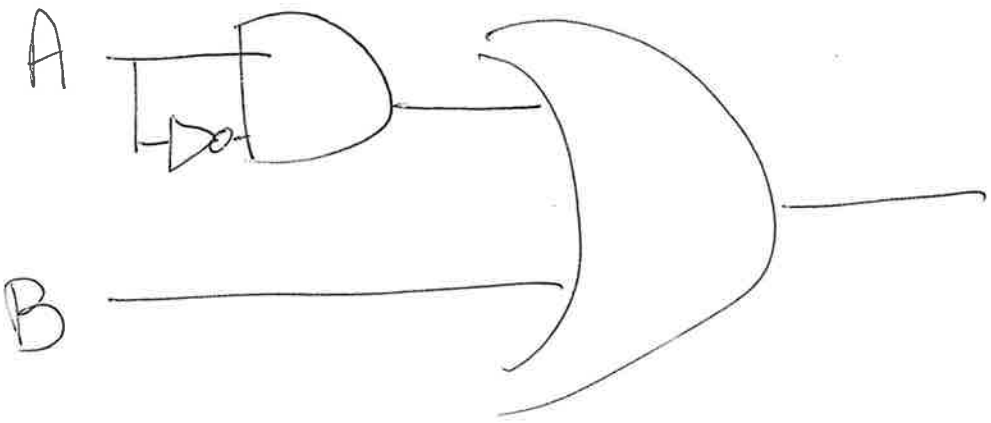
Exercice 3:



A	$\overline{A}$	O
1	0	0
0	1	0

Building a circuit with logic table:

A	B	O
1	1	1
1	0	0
0	1	1
0	0	0



(5)

You are on the island of knights and knaves. You meet John, Kai, and Tania.

John: Tania is **not** a knave

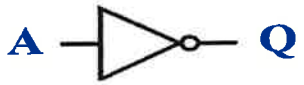
Kai: John **and** Tania are knights

Tania: John is a knight **or** Kai is a knave (inclusive).

Step 1: (K: knight, N: knave)

John	Kai	Tania	S_1	S_2	S_3	
K	K	K	T	T	T	
(K)	K	N	(F)	F	T	X
K	(N)	K	T	(T)	T	X
(K)	N	N	(F)	F	T	X
(N)	K	K	(T)	F	F	X
N	(K)	N	F	(F)	F	X
(N)	N	K	(T)	F	T	X
N	N	(N)	F	F	(T)	X

NOT



$\overline{A}$ or $\neg A$

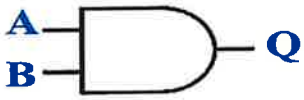
Input	Output
A	Q
1	0
0	1

OR

A+B or A∨B

Input 1	Input 2	Output
A	B	Q
1	1	1
1	0	1
0	1	1
0	0	0

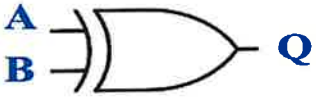
AND



A•B or A∧B

Input 1	Input 2	Output
A	B	Q
1	1	1
1	0	0
0	1	0
0	0	0

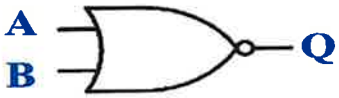
XOR



A⊕B

Input 1	Input 2	Output
A	B	Q
1	1	0
1	0	1
0	1	1
0	0	0

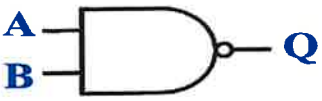
NOR



$\overline{A+B}$ or $\overline{A \vee B}$

Input 1	Input 2	Output
A	B	Q
1	1	0
1	0	0
0	1	0
0	0	1

NAND



$\overline{A \cdot B}$ or $\overline{A \wedge B}$

Input 1	Input 2	Output
A	B	Q
1	1	0
1	0	1
0	1	1
0	0	1

XNOR

$\overline{A \oplus B}$

Input 1	Input 2	Output
A	B	Q
1	1	1
1	0	0
0	1	0
0	0	1