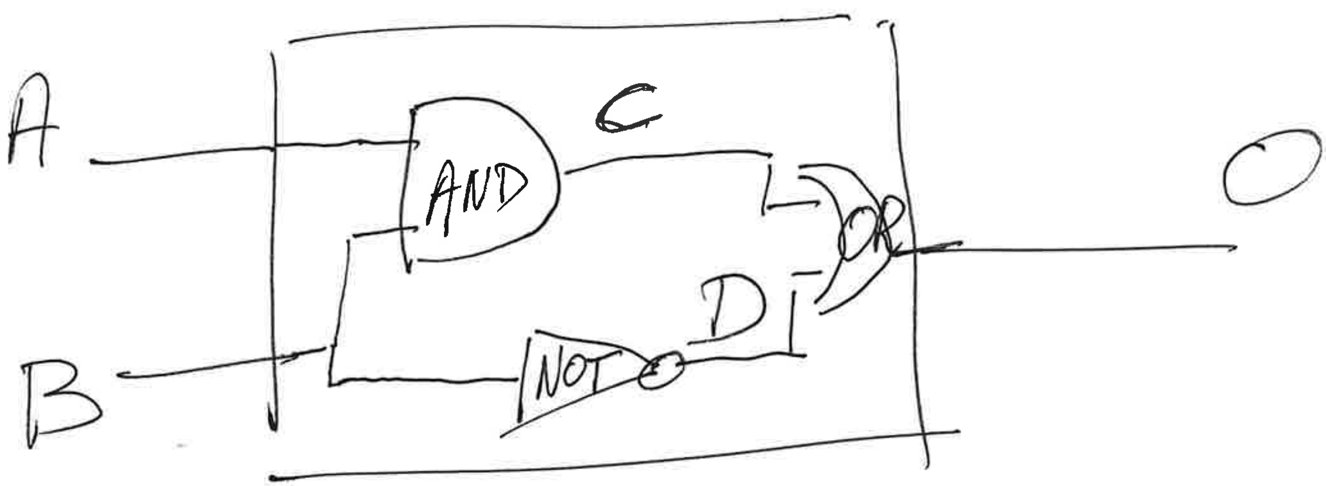
Discussion 1/28/26

Exercise 1 :



Logic table

A	B	C	D	O
1	1	1	0	1
1	0	0	1	1
0	1	0	0	0
0	0	0	1	1

Boolean expression :

$$O = C + D$$

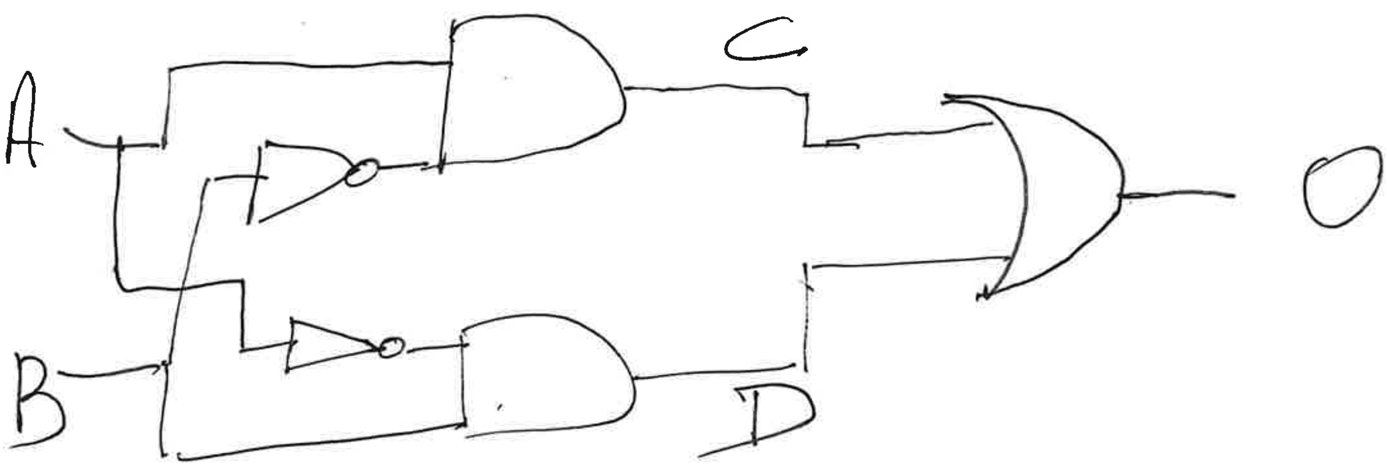
$$= A \cdot B + \overline{B}$$

Exercise 2

Build the diagram (circuit) and the logic table for the Boolean expression:

$$O = A \cdot \overline{B} + \overline{A} \cdot B$$

AND OR AND



A	B	$\overline{A}$	$\overline{B}$	C	D	O
1	1	0	0	0	0	0
1	0	0	1	1	0	1
0	1	1	0	0	1	1
0	0	1	1	0	0	0

Therefore

$$A\overline{B} + \overline{A}B = A \oplus B$$

An interesting observation:

XNOR:

$$\overline{A \oplus B} = \overline{A\overline{B} + \overline{A}B}$$

$$= \overline{A\overline{B}} \cdot \overline{\overline{A}B}$$

$$= \left(\overline{A} + \overline{\overline{B}}\right) \cdot \left(\overline{\overline{A}} + \overline{B}\right)$$

$$= \left(\overline{A} + B\right) \cdot \left(A + \overline{B}\right)$$

Exercise 3

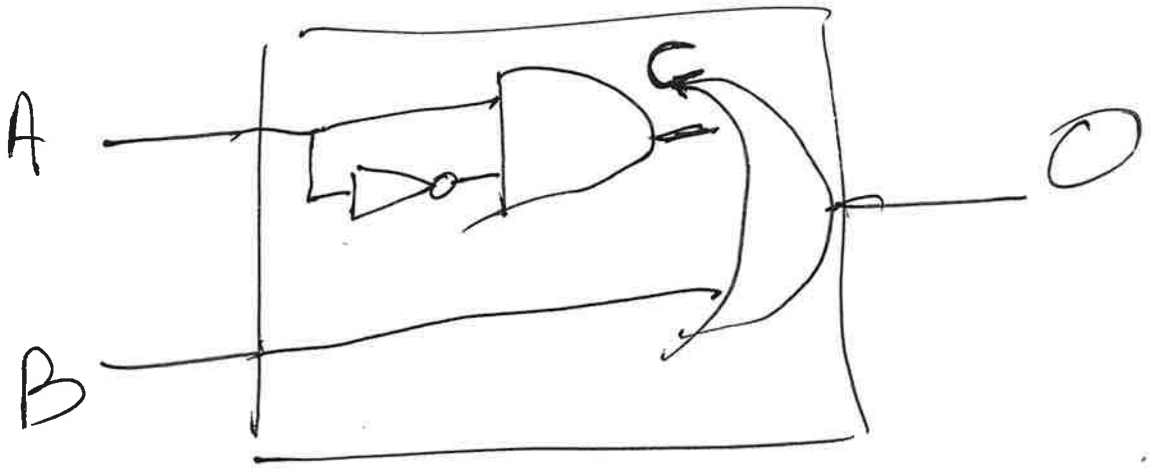
$$O = A \cdot \overline{A}$$

Logic table:

A	$\overline{A}$	O
1	0	0
0	1	0

Build a circuit with the logic table:

A	B	O
1	1	1
1	0	0
0	1	1
0	0	0



A	B	C	O
1	1	0	1
1	0	0	0
0	1	0	1
0	0	0	0

Exercise 4

To the island of knights and knaves.

You meet John, Kai, and Tania.

John: Tania is $\boxed{\text{not}}$ a knave

Kai: John $\boxed{\text{and}}$ Tania are knights

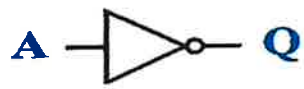
Tania: John is a knight $\boxed{\text{or}}$ Kai is a knave (inclusive).

K: knight; N: knave

	John	Kai	Tania	S_1	S_2	S_3
	K	K	K	T	T	T
X	K	K	N	F	F	T
X	K	N	K	T	T	T
X	K	N	N	F	F	T
X	N	K	K	T	F	F
X	N	K	N	F	F	F
X	N	N	K	T	F	T
X	N	N	N	F	F	T

All 3 are knights.

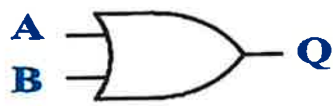
NOT



$\overline{A}$ or $\neg A$

Input	Output
A	Q
1	0
0	1

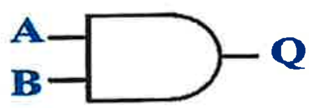
OR



$A+B$ or $A \vee B$

Input 1	Input 2	Output
A	B	Q
1	1	1
1	0	1
0	1	1
0	0	0

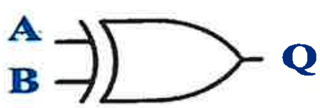
AND



$A \cdot B$ or $A \wedge B$

Input 1	Input 2	Output
A	B	Q
1	1	1
1	0	0
0	1	0
0	0	0

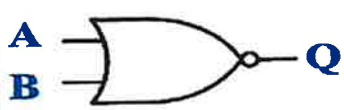
XOR



$A \oplus B$

Input 1	Input 2	Output
A	B	Q
1	1	0
1	0	1
0	1	1
0	0	0

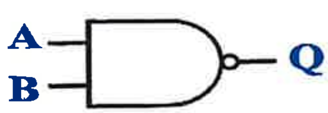
NOR



$\overline{A+B}$ or $\overline{A \vee B}$

Input 1	Input 2	Output
A	B	Q
1	1	0
1	0	0
0	1	0
0	0	1

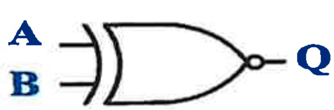
NAND



$\overline{A \cdot B}$ or $\overline{A \wedge B}$

Input 1	Input 2	Output
A	B	Q
1	1	0
1	0	1
0	1	1
0	0	1

XNOR



$\overline{A \oplus B}$

Input 1	Input 2	Output
A	B	Q
1	1	1
1	0	0
0	1	0
0	0	1