Discussion 2/2/26

Exercise 1

Show that $[p \wedge (p \to q)] \to q$ is a tautology.

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

tautology

$$
\begin{aligned}
A &\iff [p \wedge (p \to q)] \to q \\
&\iff [p \wedge (\neg p \vee q)] \to q \\
&\iff [(p \wedge \neg p) \vee (p \wedge q)] \to q \\
&\iff [F \vee (p \wedge q)] \to q \\
&\iff (p \wedge q) \to q \\
&\iff \neg (p \wedge q) \vee q \\
&\iff \neg p \vee \neg q \vee q \iff \neg p \vee T \iff T
\end{aligned}
$$

Property $p \to q \iff \neg p \vee q$

Distributivity

Exercise 2

You have a deck of cards. Each card has a letter on its front and a number (between 0 and 9) on its back.
You are told that this deck of card satisfies one property:
If you have a D on the front, then you have a 3 on the back.

You have these 4 cards in front of you:

D	E	3	7

What is the minimum number of card you need to turn to check the property?

p: D is on the front
q: 3 is on the back. | $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

When the card is D p is true. I need to check because q could be false, in which case $p \to q$ would be false.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

When the card is E
p is false, then
$p \to q$ is always true.
You don't need to turn
the card.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

The card is 3
q is true and then
$p \to q$ is always true.
You don't need to turn
the card.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

The card is 7.
q is false.
$p \to q$ could be true or
false. I need to
flip the card.

Exercise 3: Inspector Craig.

A bank in London's financial district has been robbed. The notorious criminals A, B, and C have been brought in for questioning. The following evidence was gathered:

1) A is innocent.
2) If B is guilty, then A is also guilty
3) One of the 3 (at least) is guilty

Definition of proposition:

P_A : A is guilty

P_B : B is guilty

P_C : C is guilty.

1) $\neg P_A$ is true.

2) $P_B \Rightarrow P_A$

3) $P_A \vee P_B \vee P_C$ is true.

Since $\neg p_A$ is true, p_A is false

$$\underbrace{p_B \to p_A}_{\text{true}} \iff \underbrace{\boxed{\neg p_A \to \neg p_B}}_{\text{true}}.$$

A	B	$A \to B$
T	T	T
T	F	F
F	F	T
F	F	T

Therefore $\neg p_B$ is true: p_B is false.

A and B are innocent, therefore

C is guilty : p_C is true