Discussion 2/4/26

Exercise 1:

Let p and q be 2 propositions.
Show that

$$\underbrace{[p \wedge (p \to q) \to q]}_{A}$$

is a tautology.

p	q	$p \to q$	$p \wedge (p \to q)$	A
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$$\begin{aligned}
A &\Leftrightarrow [p \wedge (p \to q)] \to q \\
&\Leftrightarrow [p \wedge (\neg p \vee q)] \to q \\
&\Leftrightarrow [(p \wedge \neg p) \vee [p \wedge q]] \to q \\
&\Leftrightarrow [F \vee (p \wedge q)] \to q \\
&\Leftrightarrow (p \wedge q) \to q \\
&\Leftrightarrow \neg (p \wedge q) \vee q \Leftrightarrow \neg p \vee \neg q \vee q \Leftrightarrow T
\end{aligned}$$

Exercise 2:

You have a deck of cards. On the front of each card, there is a letter. On the back of each card, there is a number between 0 and 9.

Property: if the front is a D, then the back is a 3.

You are shown 4 cards:

$\boxed{D}$ $\boxed{E}$ $\boxed{3}$ $\boxed{7}$

What is the minimum number of cards you need to turn to check the property?

$p \to q$: p: the front is a D
q: the back is a 3.

First card : D

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p is true, but then $p \to q$ could be false. I need to check q, meaning I need to turn the card.

Card is E

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p is false, therefore $p \to q$ is always true. I don't need to check the card.

Card is 3

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

q is true then $p \to q$ is always true. I don't ~~the~~ need to check the card.

Card is 7

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

q is false $p \to q$ may be true or false. I need to check the card.

Exercise 3 Inspector Craig.

A bank in London's financial district has been robbed. Inspector Craig has been called to find the perpetrator(s). After a few days, Inspector Craig brings 3 persons, A, B, and C for questioning. After questioning, Inspector Craig came to the conclusions that

a) A is innocent
b) If B is guilty, then A is guilty
c) At least one of the 3 is guilty.

P_A: A is innocent.
P_B: B is innocent.
P_C: C is innocent.

a) P_A is true.

b) $\neg P_B \to \neg P_A$

c) $\neg P_A \vee \neg P_B \vee \neg P_C$ is true.

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a) P_A is true

b) $\neg P_B \rightarrow \neg P_A$

In class, we stated $(p \rightarrow q) \Leftrightarrow (\neg q \rightarrow \neg p)$

$\neg q \rightarrow \neg p$ is the contrapositive of $p \rightarrow q$.

$$\neg P_B \rightarrow \neg P_A \Leftrightarrow \neg(\neg P_A) \rightarrow \neg(\neg P_B)$$

$$P_A \rightarrow P_B$$

P_A is true
$P_A \rightarrow P_B$ is true. $\Big] \rightarrow$ P_B is true.

P_A	P_B	$P_A \rightarrow P_B$	
T	T	T	$\leftarrow$ P_B is true.
~~T~~	~~F~~	~~F~~	
~~F~~	~~T~~	~~T~~	
~~F~~	~~F~~	~~T~~	

P_A and P_B are true $\rightarrow$ based on c) P_C is false.