Discussion 2/14

Exercise 1

Let n be an integer. Show that if n is odd, then $(n+1)^2$ is even.

p : n is odd $\qquad\qquad$ $\neg p$: n is even

direct $\qquad\qquad$ indirect

q : $(n+1)^2$ is even $\qquad\qquad$ $\neg q$: $(n+1)^2$ is odd.

Direct proof:

Promise: p is true.

n is odd $\qquad n = 2k+1$, k is an integer.

$$(n+1)^2 = (2k+1+1)^2 = (2k+2)^2$$
$$= [2(k+1)]^2 = 4(k+1)^2 = 2\underbrace{[2(k+1)^2]}_{\text{integer}}$$

$(n+1)^2$ is even: q is true.

By direct proof, I have shown $p \to q$ is true.

Exercise 2:

Let a and b be 2 integers.

Use a direct proof to show that

if $a^2 + b^2$ is even, then $a+b$ is even.

p : $a^2 + b^2$ is even

q: $a+b$ is even.

Direct proof:

Premise: p is true, a^2+b^2 is even

$$a^2 + b^2 = 2k, \quad k \text{ is an integer.}$$

$$a^2 + b^2 + 2ab - 2ab = 2k$$

$$(a+b)^2 = 2k + 2ab$$

$$= 2\underbrace{(k+ab)}_{\text{integer}}$$

$(a+b)^2$ is even. Then $a+b$ is even : q is true.

By direct proof, I have shown $p \to q$ is true

If a and b are 2 integers:

$a + b$ is an integer

$a - b$ is an integer

$a * b$ is an integer.

$\frac{a}{b}$ may not be an integer!

$\sqrt{a}$ may not be an integer!

Exercise 3 Let n be an integer.

Use a direct proof to show that

if $2n^3 + n + 9$ is odd, then n is even.

p: $2n^3 + n + 9$ is odd

q: n is even.

Direct proof:

Promise: p is true.

$2n^3 + n + 9$ is odd

$$\underbrace{2n^3}_{\text{even}} + n + 9 = 2k+1, \quad k \text{ integer.}$$

$2n^3$ is even: $2n^3 = 2\ell$, ℓ integer.

$$2\ell + n + 9 = 2k+1$$

$$\begin{aligned} n &= -2\ell - 9 + 2k + 1 \\ &= -2\ell + 2k - 8 \\ &= 2\,[\underbrace{-\ell + k - 4}_{\text{integer.}}] \end{aligned}$$

n is even, q is true.

By direct proof, $p \to q$ is true.

Exercise 4 Let n be an integer

Show that if $5n^2 + 8n + 13$ is odd, then n is even, using a direct proof.

p : $5n^2 + 8n + 13$ is odd

q : n is even.

Direct proof:

Premise: p is true

$5n^2 + 8n + 13$ is odd

$$5n^2 + 8n + 13 = 2k + 1, \quad k \text{ integer}.$$

$$5n^2 + 8n = 2k - 12$$

$$5n^2 = 2(k - 6 - 4n)$$

$$n^2 + 4n^2 = 2(k - 6 - 4n)$$

$$n^2 = 2[\underbrace{k - 6 - 4n - 2n^2}_{\text{integer}}]$$

n^2 is even therefore n is even; q is true.