Discussion 2/19/25

Exercise 1 Let n be an integer.

Show that if $3n$ is odd, then $(n+1)^2$ is even.

$$p \to q$$

p : $3n$ is odd $\qquad \neg p$: $3n$ is even

q : $(n+1)^2$ is even. $\qquad \neg q$: $(n+1)^2$ is odd

I will use a direct proof.

Premise: p is true.

$3n$ is odd.

There exists an integer k such that

$$3n = 2k + 1$$

$$n + 2n = 2k + 1$$

$$n = 2\underbrace{(k - n)}_{\text{integer}} + 1$$

Therefore n is odd. There exists an integer ℓ such that $n = 2\ell + 1$

$$(n+1)^2 = (2p+1+1)^2$$
$$= (2p+2)^2$$
$$= (2(p+1))^2 = 2^2 \times (p+1)^2$$
$$= 2 \times [\underbrace{2(p+1)^2}_{\text{integer}}]$$

$(n+1)^2$ is even : q is true.

By direct proof, I have shown $p \to q$ is true.

<u>Exercise 2</u> : Let <u>x be a real number different from $\emptyset$.</u>

Use a proof by contradiction to show that

if $x + \frac{1}{x} < 2$ then $x < 0$.

$p \to q$

p : $x + \frac{1}{x} < 2$ $\qquad$ $\neg p$: $x + \frac{1}{x} \geqslant 2$

q : $x < 0$ $\qquad$ $\neg q$: $x \geqslant 0$.

Proof by contradiction

We assume $(p \to q)$ is false, i.e

we assume p is true AND q is false.

p is true: $$x + \frac{1}{x} < 2 \quad (1)$$

$\neg q$ is true: $$x \geq 0 \to x > 0 \quad (2)$$

Multiply (1) by x:

$$x\left(x + \frac{1}{x}\right) < 2x$$

$$x^2 + 1 < 2x$$

$$x^2 - 2x + 1 < 0$$

$$(x-1)^2 < 0$$

This, however, is a contradiction.

Therefore the assumption $(p \to q)$ is false, is false.

By proof by contradiction, I have shown $p \to q$ is true

Exercise 3

Let a and b be 2 integers.
Show that $a^2 - 4b \neq 2$

$p: \quad a^2 - 4b \neq 2$

Proof by contradiction
I assume p is false, $a^2 - 4b = 2$

therefore $a^2 = 2 + 4b$
$= 2\underbrace{(1+2b)}_{\text{integer}}$

Therefore a^2 is even, hence a is even.
there exists an integer k such that
$a = 2k$
Replacing in the original equation,

$$(2k)^2 - 4b = 2$$

$$4k^2 - 4b = 2$$

divide by 2

$$2k^2 - 2b = 1$$

However, $2k^2 - 2b$ is even, while 1 is odd.
I have reached a contradiction.

By proof by contradiction, I have shown that p is true.

Exercise 4

Let n be a natural number.

Show that $\frac{6n+3}{4n+6}$ is not an integer.

p : $\frac{6n+3}{4n+6}$ is not an integer.

I use a proof by contradiction.

I assume p is false, $\frac{6n+3}{4n+6}$ is an integer.

Let k be that integer:

$$\frac{6n+3}{4n+6} = k$$

$$\underbrace{6n+3}_{LHS} = \underbrace{k(4n+6)}_{RHS}$$

$$LHS = 6n+3 = 6n+2+1 = 2\underbrace{[3n+1]}_{\text{Integer}} + 1$$

LHS is odd.

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$$\text{RHS} = k(4n+6)$$
$$= 2k(2n+3)$$
$$= 2[\underbrace{k(2n+3)}_{\text{integer}}]$$

RHS. is even.

I have reached a contradiction as a odd number (LHS) cannot be equal to an even number (RHS).

By proof by contradiction, I have shown p is true.