

January 8th  
Good practices

1) Problem 1:

Solve  $x^2 = 2$

$$x = +\sqrt{2} \quad \text{or} \quad x = -\sqrt{2}$$

2) Problem 2:

Let  $x$  be a real number.

Find such  $x$  that satisfies

$$\sqrt{1-x} = x+1$$

Square both sides

$$(\sqrt{1-x})^2 = (x+1)^2$$

$$1-x = x^2 + 2x + 1$$

$$0 = x^2 + 2x + \cancel{1} + x$$

$$0 = x^2 + 3x$$

$$0 = x(x+3) \quad S = \{0, -3\}$$

Problem 1:

Let  $x$  be a real number.

Show that

$$(x-1)(x+2) - x^2 + 2 = 2(x+1) - x - 2$$

Proof:

~~$$\begin{aligned} & \sqrt{(x-1)(x+2) - x^2 + 2 = 2(x+1) - x - 2} \\ \hookrightarrow & x^2 + 2x - x - 2 - x^2 + 2 = 2x + 2 - x - 2 \\ & x = x \end{aligned}$$~~

~~This is wrong as we assume that the identity  $\sqrt{\quad}$  is true but do not show that it  $\nabla$  is actually true.~~

Let us define:

$$\text{LHS}(x) = (x-1)(x+2) - x^2 + 2$$

$$\text{RHS}(x) = 2(x+1) - x - 2$$

$$\begin{aligned} \text{LHS}(x) &= (x-1)(x+2) - x^2 + 2 \\ &= x^2 + 2x - x - 2 - x^2 + 2 = x \end{aligned}$$

$$\text{RHS}(x) = 2(x+1) - x - 2 = 2x + 2 - x - 2 = x$$

Therefore  $\text{LHS}(x) = \text{RHS}(x)$

Problem 2:

Let  $n$  be a natural number.

Show that  $n(n+1) \geq n^2$

$$\text{LHS}(n) = n(n+1)$$

$$\text{RHS}(n) = n^2$$

let us compute:

$$\begin{aligned} \text{LHS}(n) - \text{RHS}(n) &= n(n+1) - n^2 \\ &= n^2 + n - n^2 \\ &= n \end{aligned}$$

$n$  is a natural number:  $n > 0$

$$\text{LHS}(n) - \text{RHS}(n) > 0$$

therefore  $\text{LHS}(n) > \text{RHS}(n)$

Problem 4:

(4)

We are on the island of knights and knaves.

We meet two persons, John and Sophie.

John: "We are both knaves"

Sophie: "I like chocolate"

John	Sophie	John says	
<del>Knight</del>	<del>Knight</del>	<del>False</del>	No: John cannot lie a knight cannot lie
<del>Knight</del>	<del>Knave</del>	<del>False</del>	
<del>Knave</del>	<del>Knight</del>	<del>False</del>	No: John, a knave cannot tell the truth.
<del>Knave</del>	<del>Knave</del>	<del>True</del>	

Sophie is a knight. Her statement is then true and she likes chocolate.

Problem 5:

(5)

A bottle of wine costs \$11.00

The wine alone costs \$10.00 more

than the bottle itself

How much costs the wine? the bottle?

Let us define:

$W$ : cost of the wine

$B$ : cost of the bottle

$$W + B = 11$$

$$W = B + 10$$

$$B + 10 + B = 11$$

$$2B = 11 - 10 = 1$$

$$B = 0.5$$

$$W = 0.5 + 10 = 10.5$$

Problem 3:

On an island there are knights and knaves. Knights always tell the truth while knaves always lie.

You meet 2 inhabitants, John and Sophie.

John: "We are of the same kind".

Sophie: "We are different".

John	Sophie	John says	Sophie says
<del>Knight</del>	<del>Knight</del>	<del>True</del>	<del>False</del>
<del>Knight</del>	<del>Knave</del>	<del>False</del>	<del>True</del>
<del>Knave</del>	<del>Knight</del>	<del>False</del>	<del>True</del>
<del>Knave</del>	<del>Knave</del>	<del>True</del>	<del>False</del>

No: Sophie could not lie