Logic 1/27/25

(Language of Mathematics)

I) Propositions

Definition: A proposition, also called statement, is a declarative sentence that is either true (T) or false (F) but not both.

Example:

Sentence	Is it a proposition?	Truth value
I wear black jeans today	Yes	True.
Today is Friday.	Yes	False (now) maybe true (later)
$2+2=5$	Yes	False
$x+1=2$	Not a proposition.	
There exists an integer x such that $x+1=2$	Yes	True

Notation:

p : "$2+2=4$"

means p is the proposition "$2+2=4$".

II) Compound propositions

Forming new propositions from old ones.

Two types of rules:

- rule that works on 1 proposition
- rules that work on 2 propositions.

II.1.) Negation.

Definition: Let p be a proposition. The sentence "it is not the case that p" is another proposition, called the negation of p, denoted $\neg p$, or sometimes $\sim p$.

It is read "not p"

if p: "$2+2=4$" True

$\neg p$: "$2+2\neq 4$" False

Truth table:

p	$\neg p$
T	F
F	T

II) . 2 ~~C~~ Conjunction

Definition: The conjunction of 2 propositions p and q is the compound proposition that we write $p \wedge q$ and read "p and q", which is only true if p is true and q is true.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

II. 3. Disjunction

Definition: The disjunction of two propositions p and q is the compound proposition written as $p \vee q$, and read "p or q", that is true if either p is true, or q is true, or both are true.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

II. 4. Exclusive or

Definition: The "exclusive or" of two proposition p and q is the compound proposition written as $p \oplus q$, and read "p xor q" that is true when p is true, q is true, but not both.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

II. 5 Comparing 2 propositions

<u>Definition</u> : Two propositions p and q are logically equivalent if they always have the same truth values.

We write $p \Longleftrightarrow q$.

<u>Exercise</u>: Let p and q be 2 propositions.

Show that

a) $\neg (p \wedge q) \Longleftrightarrow \neg p \vee \neg q$

b) $\neg (p \vee q) \Longleftrightarrow \neg p \wedge \neg q$

<u>De Morgan's laws</u>

Solving a) means we want to show $\neg (p \wedge q) \Longleftrightarrow \neg p \vee \neg q$ which means we want to show that 2 propositions are logically equivalent.

Let us define:

$A = \neg(p \wedge q)$

$B = \neg p \vee \neg q$

p	q	$p \wedge q$	$A = \neg(p \wedge q)$	$\neg p$	$\neg q$	$B = \neg p \vee \neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

always have the same truth values therefore $A \Leftrightarrow B$.

Show that $\neg(\neg p) \Leftrightarrow p$

p	$\neg p$	$\neg(\neg p)$
T	F	T
F	T	F

always have the same truth values therefore $p \Leftrightarrow \neg(\neg p)$