Logic 1/28/26

(the language of mathematics)

I) Vocabulary: the proposition.

Definition: A proposition, also called a statement, is a declarative sentence that is either true (T) or false (F) but not both.

Example:

Sentence	Is it a proposition?	Truth value
I wear black jeans today.	Yes	Yes: True
It rained yesterday.	Yes	Yes: True.
$2+2=5$	Yes	False
$x+2=4$	Not.	

x is a real number.

$$x + 2 = 4$$

There exists a real number x such that $x + 2 = 4$.

For all real numbers x,

$$x + 2 = 4.$$

The concepts of associated with "there exists" and "for all" are at the core of mathematical problems.

Notation: there exists $\to$ $\exists$

For all: $\forall$

Two fundamental propositions:

proposition that is always true is a tautology: T

A proposition that is always false is a contradiction: F

II) Grammar: Combining propositions

Two types of rules:

- rules that work on one proposition
- rules that work on 2 propositions

II. 1) Negation

Definition Let p be a proposition. The sentence "it is not the case that p" is another proposition, called the negation of p, denoted $\neg p$, or sometimes $\sim p$.

A rule is defined with a truth table:

p	$\neg p$
T	F
F	T

$\boxed{\neg T \equiv F}$

II) 2) Rules over 2 propositions

a) AND: conjunction

symbole : $p \wedge q$

table of truth:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

b) OR: disjunction

symbol $p \vee q$

truth table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR:

$p \oplus q$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

III) Comparing propositions

Two propositions are "equal" (we prefer to say logically equivalent"), if they always have the same truth values

We write $p \Leftrightarrow q$.

Example: I claim $\underbrace{\neg(\neg p)}_{A} \Leftrightarrow \underbrace{p}_{B}$

p	$\neg p$	$\neg(\neg p)$
T	F	T
F	T	F