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Logic 1/30/26
(The language of mathematics)

I) A Language

Vocabulary: propositions.

Perform operations on propositions:

$p \xrightarrow{\text{operation negation}}$

$\neg p$ ($\sim p$)

On two propositions:

AND $p \wedge q$

OR $p \vee q$

XOR: $p \oplus q$

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \oplus q$
T	T	F	T	T	F
T	F	F	F	T	T
F	T	T	F	T	T
F	F	T	F	F	F

Some properties

②

$$① \neg(\neg p) \Leftrightarrow p$$

De Morgan's laws

$$② \begin{cases} \neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q \\ \neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q \end{cases}$$

③ Distributivity:

$$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$$

II) the conditional

Definition: Given 2 propositions p and q , the conditional of p and q , written $p \rightarrow q$ (p implies q) is a new proposition that is false if p is true and q is false, and true otherwise.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example: A conditional is a "contract".

"I you cheat during an exam, you get a 0"
This is a contract.

The contract is broken if you cheat,
and I don't give you a 0.

Some interesting properties about
implications: ✓

$$a) (p \rightarrow q) \Leftrightarrow (\neg p \vee q)$$

$$b) (p \rightarrow q) \Leftrightarrow (\neg q \rightarrow \neg p)$$

$p \Leftrightarrow q$: $(\Leftrightarrow)$ is a comparison
operator,
logical equivalence.

$$(p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p))$$

biconditional

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T