Logic 1/31

The language of mathematics.

① Proposition.

② Operators on proposition.

Not : $\neg p$

And : $p \wedge q$

or : $p \vee q$ (inclusive)

$p \oplus q$ (exclusive)

conditional $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Exercise:

Show that

$A = (p \wedge q) \vee (\neg p \vee \neg q)$ is a tautology

1) safe method

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	A
T	T	T	F	F	F	T
T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

2) $B = p \wedge q$

$\neg B = \neg(p \wedge q) = \neg p \vee \neg q$

$A = B \vee \neg B \Leftrightarrow T$

Exercise : proof

Show that $\boxed{(p \to q) \wedge (q \to p) \Leftrightarrow p \leftrightarrow q}$

Define $A = (p \to q) \wedge (q \to p)$

$B = p \leftrightarrow q$

p	q	$p \to q$	$q \to p$	A	B
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

have the same truth values.
They are logically equivalent.

Show that $(p \wedge q) \vee (\neg p \vee \neg q)$ is a tautology.

1) Note that $A \vee \neg A$ is a tautology.

A	$\neg A$	$A \vee \neg A$
T	F	T
F	T	T

2) Let $A = (p \wedge q) \vee (\neg p \vee \neg q)$

Notice that $\neg p \vee \neg q \Leftrightarrow \neg (p \wedge q)$

$A \Leftrightarrow (p \wedge q) \vee [\neg (p \wedge q)]$.

$A \Leftrightarrow \underbrace{B \vee [\neg B]}$

$\Leftrightarrow$ is a tautology.

Quantifiers

Take the sentence $x + 4 = 6$.

Is this a proposition? No,

for 2 reasons:

Reason 1: I miss a concept of domain. $D = \mathbb{Z}$.

Ambiguity:

Did I mean:

For all integers x, $x + 4 = 6$

$\rightarrow$ becomes a proposition, False

or

There exists an integer x, $x + 4 = 6$.

$\rightarrow$ becomes a proposition, true

$x = 2$.

For all: $\forall$

There exists: $\exists$