Proofs 2/16

I) Proof by contradiction

If we want to show that a proposition p is true, we can instead assume that $\neg p$ is true, and hope to reach a contradiction. If this is the case, i.e we show $\neg p \to F$ then necessarily p is true.

Example: $\sqrt{2}$ is irrational.

Can that work for implication?

An implication A is a proposition that I can write as $A \Leftrightarrow (p \to q)$

A proof by contradiction would assume $\neg A$ is true.

$$\neg(p \to q) \text{ is true}$$

$$\neg(\neg p \vee q) \text{ is true}$$

$$p \wedge \neg q \text{ is true.}$$

Example: Let x and y be 2 real numbers.

Show that if $2x+4y+1=2024$ then x or y is not an integer.

p: $2x+4y+1=2024$

q: x or y is not an integer

$\neg p$: $2x+4y+1 \neq 2024$

$\neg q$: x AND y are integers.

I attempt a proof by contradiction.

I assume p is true AND q is false.

p: $\underbrace{2x + 4y + 1}_{LHS} = \underbrace{2024}_{RHS}$

$\neg q$: x and y are integers.

$$LHS = 2\underbrace{(x + 2y)}_{\text{integer}} + 1 \equiv \text{odd}$$

$$RHS = 2024 \equiv \text{even}.$$

However, an odd number cannot be equal to an even number.

Biconditional

We spent a fair amount to time to define methods to prove implications: $p \to q$ (If p then q)

What if I want to prove a biconditional?

$$p \Longleftrightarrow q$$

p if and only if q

$$p \Leftrightarrow q \Leftrightarrow (p \to q) \wedge (q \to p)$$

Proofs and quantifiers

Let n be an integer.

Show that if n is odd, then n^2+1 is even.

n	n^2+1	
1	2	even
3	10	even
5	26	even
7	50	even
$\vdots$	$\vdots$	

~~True~~.

This is not valid, as a proof by examples only work if there is a finite set of examples.

"Let n be an integer"

really means

for all n integers

$\forall \ n \in \mathbb{Z}$

Example:

Let n be an ~~integer~~ positive integer.

Prove or disprove that $2^n + 1$ is prime.

n	2^n+1
0	$2^0+1 = 2$ prime
1	$2^1+1 = 3$ prime
2	$2^2+1 = 5$ prime
3	$2^3+1 = 9$ Not prime.

I found a counter example.

P is not true.

Example:

Show that either $2^{2024}+5^{1012}+7^{512}+16$ or $2^{2024}+5^{1012}+7^{512}+17$ is even.

$$a = 2^{2024}+5^{1012}+7^{512}+16$$
$$b = 2^{2024}+5^{1012}+7^{512}+17$$

$$b = a+1$$

Since a and b are consecutive, one of them is even. I don't know which one → non constructive proof