Proof by induction

1. Theorem:

Let $P(n)$ be a property defined for a natural number n.

If I can show that:

$P(n_0)$ is true (basic step)

$P(n) \rightarrow P(n+1)$ is true for $n \geq n_0$ (inductive step)

then I can say that $P(n)$ is true for all $n \geq n_0$.

2. Example:

$$P(n): \quad 1 + 2 + \cdots n = \frac{n(n+1)}{2}$$

let $P(n): \underbrace{1+3+5+\cdots 2n-1}= \underline{n^2}$

$LHS(n) = 1+3+\cdots 2n-1$

$RHS(n) = n^2$

$P(n): \quad LHS(n) = RHS(n)$

I want to show $P(n)$ is true, $\forall n \geq 1$.

Step 1: Basic step. $n=1$

$LHS(1) = 1$

$RHS(1) = 1^2 = 1$

$\Rightarrow LHS(1) = RHS(1)$

$P(1)$ is true.

Step 2: Inductive step:

I want to show $P(n) \to P(n+1)$, $n \geq 1$.

Direct proof: $P(n)$ is true, i.e.,

$LHS(n) = RHS(n)$

$LHS(n+1) = \underbrace{1+3+5+\cdots 2n-1} + 2n+1$

$= LHS(n) + 2n+1 = RHS(n) + 2n+1 = n^2+2n+1$

$RHS(n+1) = (n+1)^2 = n^2+2n+1$

Therefore $LHS(n+1) = RHS(n+1)$, $P(n+1)$ is true.

The method of proof by induction allows me to conclude that $P(n)$ is true for all $n \geq 1$.

Careful:

$P(n)$: For any groups of n bunnies, the n bunnies have the same color.

Proof by induction:

Basic step: $n = 1$

One bunny has 1 color. $P(1)$ is true.

Inductive step: $P(n) \to P(n+1)$ for all $n \geq 1$.

$P(n)$ is true: $\{B_1, \dots B_n\}$

$P(n+1)$? $\{B_1, \dots B_n, B_{n+1}\}$

$\cancel{\{B_1, \dots B_n\}}$

$P(n+1)$? $S = \{B_1, \ldots B_{n+1}\}$

$\{B_1, \ldots B_n\}$ $\therefore$ all have the same color: let us say white.

$\underbrace{\{B_2, \ldots B_{n+1}\}}$: n bunnie. They all have the same color: C

(is a set only if $n+1 > 2$)

B_2 : belongs to the first group $\to$ white.

B_2 : belongs to the second group $\to$ C

Therefore C is white.

$\{B_1, \ldots B_{n+1}\}$ have the same color.

$P(n+1)$ is true.

The method of proof by induction allows me to conclude that $P(n)$ is true for all $n \geq 1$

(All bunnies are white).