Fibonacci 2/23/16

Fibonacci is bunnies

time

of (pairs) of bunnies

0

$F_0 = 0$

1 bb

$F_1 = 1$

2 BB

$F_2 = 1$

3 BB bb

$F_3 = 2$

4 BB bb BB

$F_4 = 3$

5 BB bb BB BB bb

$F_5 = 5$

6 BB bb BB BB bb BB bb BB

$F_6 = F_5 + F_4$

$= 5 + 3$

What happen at month n?

Call numbers of (pairs of) bunnies as F_n

At month n:

$$F_n = F_{n-1} + F_{n-2}$$

To fully describe the Fibonacci numbers, you need to provide:

- a basis step: $F_0 = 0$, $F_1 = 1$
- an inductive step: $F_n = F_{n-1} + F_{n-2}$

$$F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right)$$

Some examples:

Exercise 1 Let F_n be the Fibonacci numbers. Show that for all $n \geq 1$

$$F_1^2 + F_2^2 + \cdots F_n^2 = F_n F_{n+1}$$

let $A(n) = F_1^2 + F_2^2 + \cdots F_n^2$

$B(n) = F_n F_{n+1}$

Let $P(n)$: $A(n) = B(n)$. I want to show $P(n)$ is true for all $n \geq 1$.

We use a proof by induction:

Basis step is $P(1)$ true?

$A(1) = F_1^2 = 1$

$B(1) = F_1 F_2 = 1 \times 1 = 1$ $\Rightarrow$ $P(1)$ is true.

Inductive step: $P(n) \rightarrow P(n+1)$ when $n \geq 1$

We need to prove $P(n) \rightarrow P(n+1)$

using a direct proof.

We assume $P(n)$ is true, therefore

$$A(n) = B(n).$$

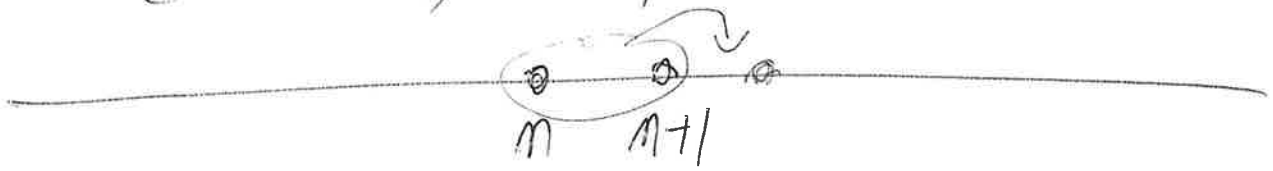
$$A(n+1) = \underbrace{F_1^2 + F_2^2 + \cdots F_n^2}_{} + F_{n+1}^2$$

$$= A(n) + F_{n+1}^2$$

$$= B(n) + F_{n+1}^2$$

$$= F_n F_{n+1} + F_{n+1}^2$$

$$= (F_n + F_{n+1}) F_{n+1}$$



$$A(n+1) = F_{n+2} F_{n+1}$$

$$B(n+1) = F_{n+1} F_{n+1+1} = F_{n+1} F_{n+2}$$

$$(B(n) = F_n F_{n+1})$$

Therefore $A(n+1) = B(n+1)$: $P(n+1)$ is true.

The method of proof by induction allows me to conclude that $P(n)$ is true for $n \geq 1$.

$$A(n) = F_1^2 + \cdots - \cdots F_n^2$$

$$= \sum_{i=1}^{n} F_i^2$$