Induction 2/23

I) Blue eye problem

$P(n)$: n people with blue eyes will leave on then n-th day.

$P(1)$ is true.

$P(2)$ is true.

$P(3)$ is true

⋮

Assume that $P(n)$ is true

What happens for $P(n+1)$?

I have $(n+1)$ student with blue eyes.

Take 1: S_1 He will see N students with blue eyes $\Rightarrow$ he will then know that he has blue eyes.

This means $P(n+1)$ is true

Assume $P(n)$ is true and show $P(n+1)$ is true.

This is a direct proof of

$$P(n) \rightarrow P(n+1).$$

Reasoning: Given a property $P(n)$, where n is a positive integer.

If I can show

$P(1)$ is true

$P(n) \rightarrow P(n+1)$ is true, for all $n \geq 1$

Then

$P(n)$ is true for all n positive integers

This is the method of proof by induction.

Example:

Let n be a natural number.

Show that

$$\underbrace{1+2+\cdots\cdots n}_{\sum_{i=1}^{n} i} = \frac{n(n+1)}{2}$$

Proof by induction:

Let us define:

$$LHS(n) = 1+2+\cdots\cdot n$$

$$RHS(n) = \frac{n(n+1)}{2}$$

I need to prove:

$$P(n): \quad LHS(n) = RHS(n)$$

is true for all $n \geq 1$;

n natural number.

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I use a proof by induction:

<u>Basis step</u> is $P(1)$ true?

$LHS(1) = 1$

$RHS(1) = \frac{1 \times (1+1)}{2} = 1$

$\Rightarrow P(1)$ is true.

<u>Inductive step</u>:

I want to show $P(n) \to P(n+1)$, for all $n \geq 1$.

I assume $P(n)$ is true.

I know: $LHS(n) = RHS(n)$

$$LHS(n+1) = \underbrace{1+2+\cdots\cdots n}+n+1$$

$$= LHS(n) + n+1$$

$$= RHS(n) + n+1$$

$$= \frac{n(n+1)}{2} + \frac{2(n+1)}{2}$$

$$= \frac{n(n+1)+2(n+1)}{2} = \frac{(n+1)(n+2)}{2}$$

$$LHS(n+1) = \frac{(n+1)(n+2)}{2}$$

$$RHS(n+1) = \frac{(n+1)(n+1+1)}{2} = \frac{(n+1)(n+2)}{2}$$

Therefore $LHS(n+1) = RHS(n+1)$

$P(n+1)$ is true.

The method of proof by induction allows me to conclude that $P(n)$ is true for all natural numbers n.