Proofs 2/6/25

I) Definitions

A theorem is a proposition that can be shown to be always true.

How to prove a theorem?

A demonstration: We demonstrate that a theorem is true with a sequence of propositions that forms an "argument", also called a proof:

A proof is:

Premise ← what you are given

Body

Conclusion.

The main proofs we are going to work on are:

- implications: $p \to q$

  If ( ) then ( )

- biconditional $p \Leftrightarrow q$

  ( ) if and only if ( )

$$p \Leftrightarrow q \iff (p \to q) \wedge (q \to p)$$

II) Two essential theorems for ECS 17:

<u>Theorem</u> 1: An integer number n is even if and only if there exists an integer k such that $n = 2k$.

Example: 12 is even, $12 = 2 \times \underbrace{6}_{\text{integer}}$.

$3 = 2 \times \left(\frac{3}{2}\right)$

Theorem 2: An integer number n is odd if and only if there exists a integer k such that

$$n = 2k+1.$$

III Proof of conditional.

Example:

If n is an even integer, then n^2 is an even integer.

p: n is an even integer

q: n^2 is an even integer.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

<u>P is true:</u>

n is an even integer.

There exists an integer k such that $n = 2k$.

$$n^2 = 4k^2$$

$$n^2 = 2\underbrace{[2k^2]}_{\text{integer}}$$

Therefore n^2 is even. <u>q is true</u>