Proofs 2/7/25

How a proof works:

Premises

Body

Conclusion.

Statements

$\downarrow$ Infer

Statements

Rule of inference: Modus ponens

$$\begin{array}{ll} p & \text{true} \\ p \to q & \text{true} \\ \hline q & \text{true.} \end{array}$$

II) Validating conditionals $p \to q$

Definition 1 An integer number n is even if and only if there exists an integer number k such that $n = 2k$

Definition 2: An integer number n is odd if and only if there exists an integer number k such that $n = 2k+1$

Proving $p \to q$ is true

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

1) Some implications are obviously true.

Let x be a real number

if $\underbrace{x^2 < -1}_{\text{False}}$ then $x + 1 = 0$

if $x^2 < -20$ then Trump will leave office tomorrow

2) Direct proofs:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

When p is true, $p \to q$ is only true if q is also true.

Direct proof of $p \to q$ works by assuming that p is true and then validating that q is also true.

Example: Let n be a natural number. Show that if n is even, then n^2 is even.

p: n is even

q: n^2 is even.

Direct proof: my premise is that p is true. Therefore n is even. There exists an integer k such that $n = 2k$

what about q? I want to show n^2 is even.

$$n^2 = (2k)^2$$
$$= 4k^2$$
$$= 2\,[\underbrace{2k^2}_{\text{integer}}]$$

Therefore n^2 is even: q is true.

I have shown that when p is true, q is also true: $p \to q$ is true.

Indirect proof

Example: Let n be an integer.
Show that if n^2 is even, then n is even.

Try a direct proof:

~~Premise:~~ ~~p~~ let p: n^2 is even
q: n is even

Premise: p is true, i.e. n^2 is even.
There exists an integer k such that $n^2 = 2k$.
What about q?

$$n^2 = 2k$$
$$\hookrightarrow n = \sqrt{2}\sqrt{k}$$

This does not work as $\sqrt{\ }$ is not guaranteed to give integers.

p: n^2 is even
q: n is even

$p \to q$ is true?

However, we showed that

$$\boxed{p \to q \iff \neg q \to \neg p}$$

$\neg q$: n is odd
$\neg p$: n^2 is odd.

Promise: $\neg q$ is true, meaning $n = 2k+1$ where k is an integer.

$$n^2 = (2k+1)^2$$
$$= 4k^2 + 4k + 1$$
$$= 2\underbrace{(2k^2 + 2k)}_{\text{integer}} + 1$$

n^2 is odd.
I have shown $\neg q \to \neg p$ is true,
therefore $p \to q$ is true.