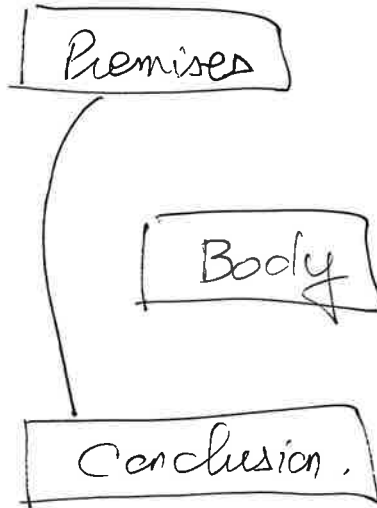


Proof

2/7/25

①

How a proof works:



Statements

↓ Infer
Statements

Rule of inference: Modus ponens

$$\begin{array}{l} P \quad \text{true} \\ \hline P \rightarrow Q \quad \text{true} \\ \hline Q \quad \text{true} \end{array}$$

II) Validating conditionals $p \rightarrow q$

Definition 1 An integer number n is even if and only if there exists an integer number k such that $n = 2k$

Definition 2: An integer number n is odd if and only if there exists an integer number k such that $n = 2k + 1$

Proving $p \rightarrow q$ is true

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

1) Some implications are obviously true. (3)

Let x be a real number

if $x^2 < -1$ then $x + 1 = 0$
False

if $x^2 < -20$ then Trump
will leave office
tomorrow

2) Direct proofs:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

When p is true, $p \rightarrow q$ is only
true if q is also true.

(4)

Direct proof of $p \rightarrow q$ works by assuming that p is true and then verifying that q is also true.

Example: Let n be a natural number.

Show that if n is even, then n^2 is even.

p : n is even

q : n^2 is even.

Direct proof: my premise is that p is true. Therefore n is even.

There exists an integer k such that $n = 2k$.
What about q ? I want to show n^2 is even.

$$\begin{aligned} n^2 &= (2k)^2 \\ &= 4k^2 \\ &= 2 [2k^2] \\ &\quad \text{integer.} \end{aligned}$$

Therefore n^2 is even: q is true.

I have shown that when p is true, q is also true: $p \rightarrow q$ is true. (5)

Indirect proof

Example: Let n be an integer.

Show that if n^2 is even, then n is even.

Try a direct proof:

Premise: let p : n^2 is even
 q : n is even

Premise: p is true, i.e. n^2 is even.

There exists an integer k such that $n^2 = 2k$.

What about q ? $n^2 = 2k$
 $\hookrightarrow n = \sqrt{2} \sqrt{k}$

This does not work as

$\sqrt{\quad}$ is not guaranteed to give integers.

(6)

p : n^2 is even
 q : n is even

$p \rightarrow q$ is true?

However, we showed that

$$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$$

$\neg q$: n is odd

$\neg p$: n^2 is odd.

Premise: $\neg q$ is true, meaning $n = 2k+1$
 where k is an integer.

$$\begin{aligned} n^2 &= (2k+1)^2 \\ &= 4k^2 + 4k + 1 \\ &= 2(\underbrace{2k^2 + 2k}_{\text{integer}}) + 1 \end{aligned}$$

n^2 is odd.

I have shown $\neg q \rightarrow \neg p$ is true,
 therefore $p \rightarrow q$ is true.