Counting 3/10

Property 1: product rule

$$|A_1 \times A_2| = |A_1| \times |A_2|$$

Property 2: sum rule (inclusion - exclusion)

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Property 3: The complement rule

$$|\bar{A}| = |D| - |A|$$

Example 1

How many words of 4 characters are there, where characters are taken from the English alphabet?

$$\boxed{26} \times \boxed{26} \times \boxed{26} \times \boxed{26} \qquad 26^4$$

Example 2: How many words of 4 English characters are there, with no repeating character.

$$\boxed{26} \times \boxed{25} \times \boxed{24} \times \boxed{23} \qquad 26 \times 25 \times 24 \times 23$$

Example 3: How many words with 4 English characters contain at least one a.

P: contains at least one a.

$\neg P$: does not contain any a.

D: all words

A: satisfy P

$\overline{A}$: satisfies $\neg P$

$$|D| = 26^4$$

$$|\overline{A}| = 25^4$$

$$|A| = |D| - |\overline{A}| = 26^4 - 25^4$$

Example 4 :

How many passwords are there that contain letters and digits, with at least one digit.

P : contains at least one digit

$\neg P$: do not contain any digits.

D : passwords

A : passwords that satisfy P

$\overline{A}$: passwords that satisfy $\neg P$.

D : $\boxed{36} \times \boxed{36} \times \boxed{36} \times \boxed{36} \times \boxed{36} \times \boxed{36}$

$$|D| = 36^6$$

$\overline{A}$: $\boxed{26}\ \boxed{26}\ \boxed{26}\ \boxed{26}\ \boxed{26}\ \boxed{26}$

$$|\overline{A}| = 26^6$$

$$|A| = 36^6 - 26^6$$

Example 5

How many bitstrings of length 8 either start with 1 or end with $\emptyset\emptyset$

S_1: bitstrings of length 8 that start with 1

S_2: bitstrings of length 8 that end with $\emptyset\emptyset$

$$S = S_1 \cup S_2$$

$$|S_1 \cup S_2| = |S_1| + |S_2| \cancel{+} - |S_1 \cap S_2|$$

S_1: $\boxed{1}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{}$

$$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$$

S_2: $\boxed{}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{\emptyset}\ \boxed{\emptyset}$

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1 \times 1 \quad 2^6$$

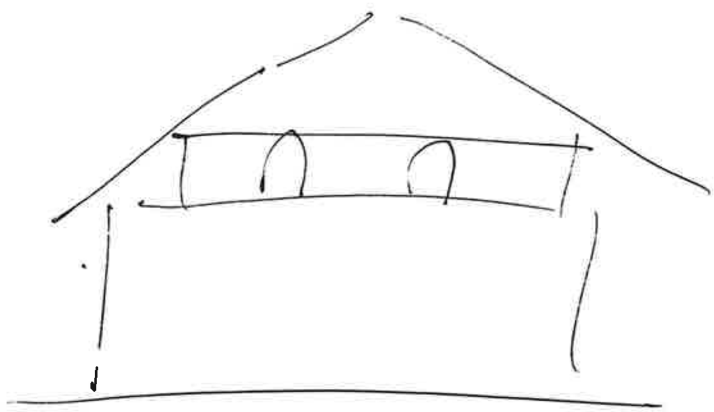
$S_1 \cap S_2$: $\boxed{1}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{}\ \boxed{\emptyset}\ \boxed{\emptyset}$

$$1 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 1 \quad 1 \qquad 2^5$$

$$|S_1 \cup S_2| = 2^7 + 2^6 - 2^5$$

The pigeonhole principle:

If $k+1$ objects (or more) are placed into k boxes, then there is at least one box that contains 2 or more elements.



Example: If you take 6 distinct numbers between 1 and 10, you are guaranteed that at least 2 of them will have a sum of 11.

1, 10	2, 9	3, 8	4, 7	5, 6
11	11	11	11	11