Counting 3/11

I) The product rule

$$|A_1 \times A_2| = |A_1| \times |A_2|$$

Example: How many 4-letter words are there if each letter is chosen from the English alphabet (no case)

$$26 \times 26 \times 26 \times 26 = 26^4$$

II) The sum rule

If a list L to be counted can be divided into two sublists L_1 and L_2 of size n_1 and n_2, respectively, and L_1 and L_2 have n common elements, then the list L contains $n_1 + n_2 - n$ elements.

$$|L| = |L_1 \cup L_2| = |L_1| + |L_2| - |L_1 \cap L_2|$$

Rule of complement:

If a list L has n objects, and m of these have a given property P, then the number of objects in L that do not have this property P is

$n - m$.

$$|\bar{A}| = |D| - |A|$$

Example: How many bit strings of length 8 either start with a 1, or end with 00?

L_1: bit strings of length 8 that starts with 1

L_2: bit strings of length 8 that ~~sta~~ end with $\emptyset\emptyset$

$$L = L_1 \cup L_2$$

$$|L| = |L_1| + |L_2| - |L_1 \cap L_2|$$

L_1 $\boxed{1}\ \square\ \square\ \square\ \square\ \square\ \square\ \square$

$$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$$

L_2 $\square\ \square\ \square\ \square\ \square\ \square\ \boxed{0}\ \boxed{0}$

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1 \quad 1 = 2^6$$

$L_1 \cap L_2$: $\boxed{1}\ \square\ \square\ \square\ \square\ \square\ \boxed{0}\ \boxed{0}$

$$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1 \times 1 = 2^5$$

$$|L| = 2^7 + 2^6 - 2^5$$

<u>Example 2</u>: How many 4-letter word whose characters are taken from the English alphabet contain at least one a ?

P: contains at least one a

$\neg P$: contains no a.

A: words that satisfy P

$\overline{A}$: words that satisfy $\neg P$

$$|\overline{A}| = |D| - |A|$$

D ?

$$\square \quad \square \quad \square \quad \square$$

$$26 \times 26 \times 26 \times 26 = 26^4$$

$\overline{A}$?

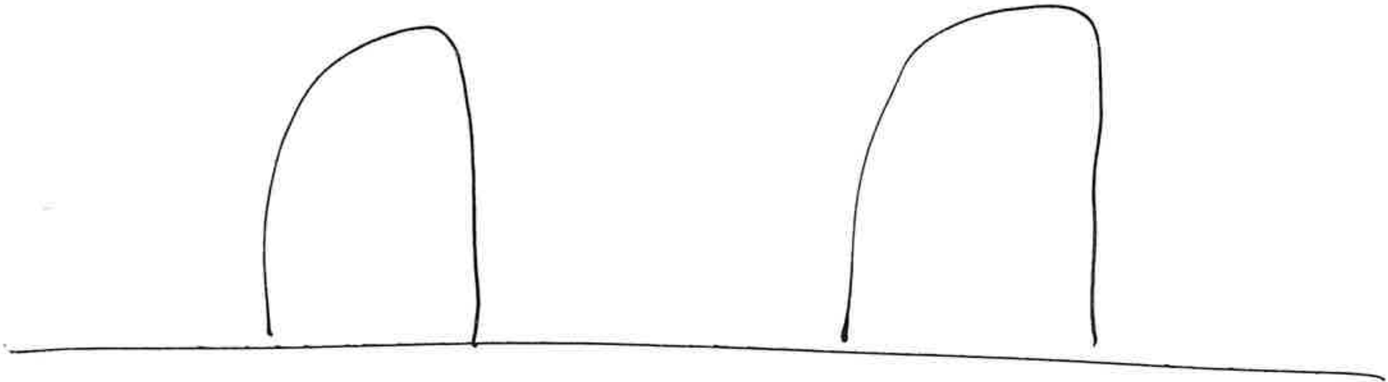
$$\square \quad \square \quad \square \quad \square$$

$$25 \times 25 \times 25 \times 25 = 25^4$$

$$|A| = |D| - |\overline{A}|$$

$$= 26^4 - 25^4$$

The pigeonhole principle



Property: If $k+1$ objects (or more) are placed into k boxes, then there is at least one box that contains 2 or more elements.