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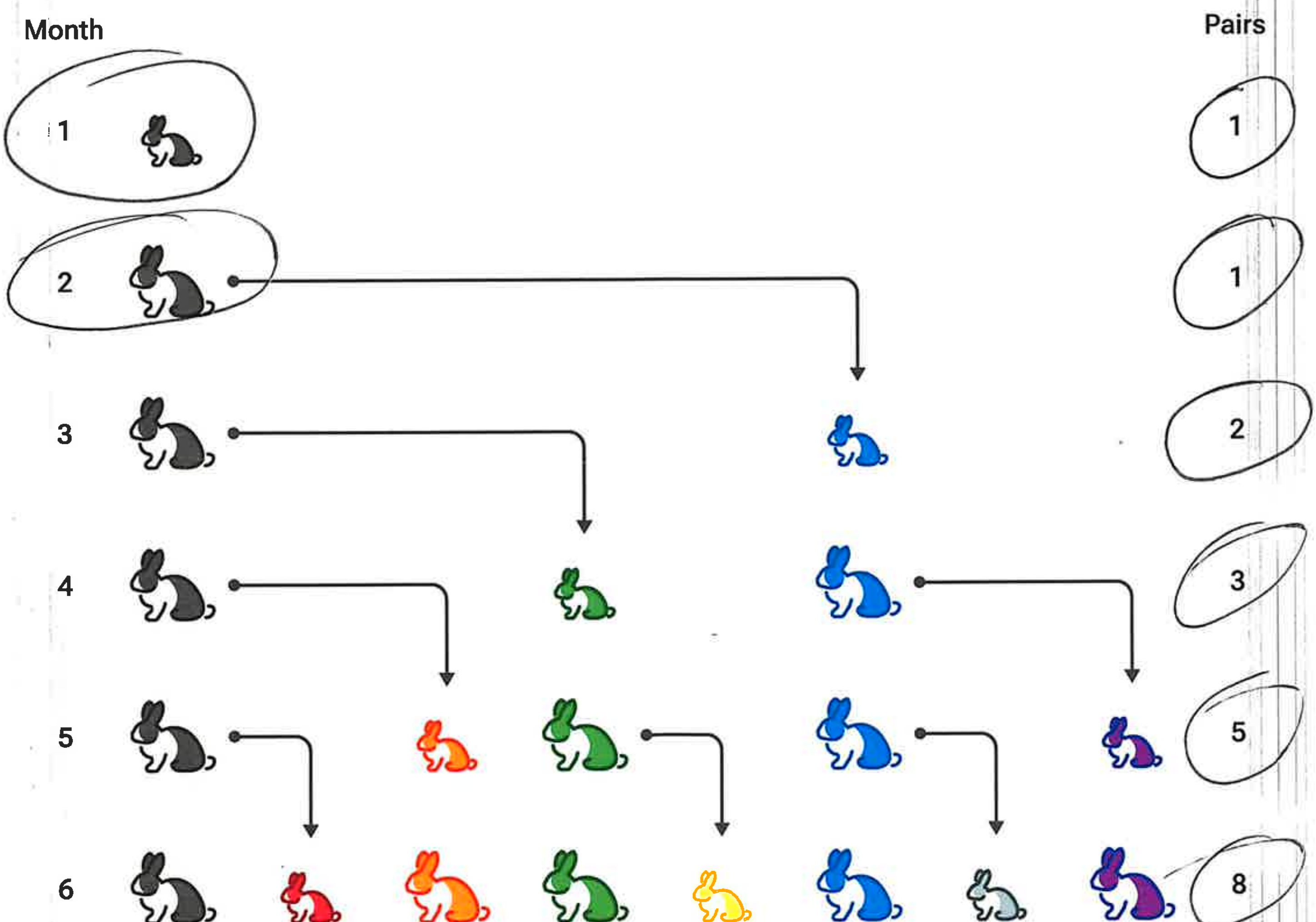


$$\boxed{f_n = f_{n-1} + f_{n-2}}$$

Fibonacci 3/4

I) Fibonacci series of number.

Fibonacci numbers are defined as:

$$\begin{cases} F_\varnothing = 0 \\ F_1 = 1 \\ F_n = F_{n-1} + F_{n-2} \quad n \geq 2 \end{cases}$$

$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

Problem 1 : Let F_n be the numbers in the Fibonacci sequence. Show that:

$$F_1 + F_3 + F_5 + \cdots F_{2n-1} = F_{2n}$$

for all $n \geq 1$

Definitions:

$A(n) = F_1 + F_3 + \cdots F_{2n-1}$

$B(n) = F_{2n}$

$P(n)$: $A(n) = B(n)$

We want to show $P(n)$ is true for all $n \geq 1$.

I use a proof by induction.

Basis step: Is $P(1)$ is true?

$A(1) = F_1 = 1$

$B(1) = F_2 = F_1 + F_0 = 1 + 0 = 1$

$A(1) = B(1)$

$P(1)$ is true.

Inductive step: $P(n) \to P(n+1)$ is true for $n \geq 1$ 3

I assume $P(n)$ is true: $A(n) = B(n)$

$$A(n+1) = \underbrace{F_1 + F_3 + \cdots\cdots F_{2n-1}} + F_{2n+1}$$

$$= A(n) + F_{2n+1}$$

$$= B(n) + F_{2n+1}$$

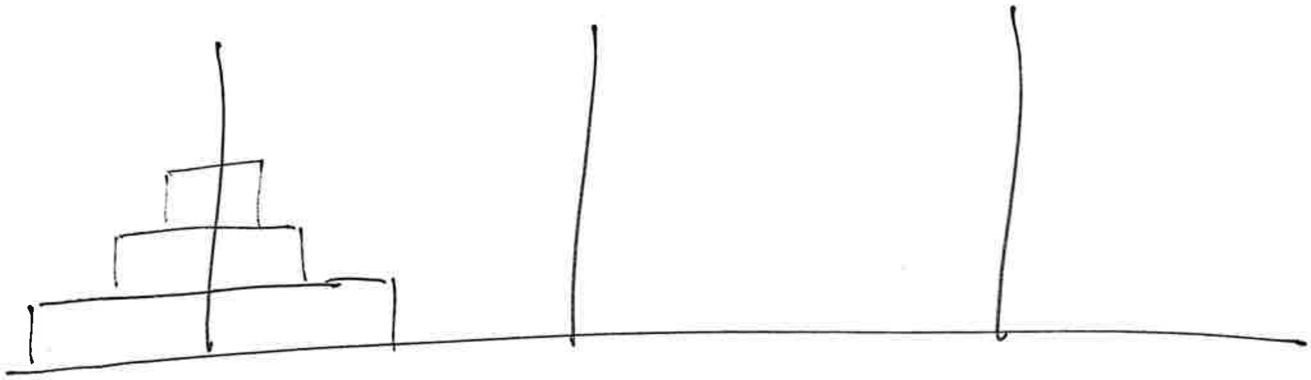
$$= F_{2n} + F_{2n+1} = F_{2n+2}$$

$$B(n+1) = F_{2(n+1)} = F_{2n+2}$$

Therefore $A(n+1) = B(n+1)$: $P(n+1)$ is true.

The method of proof by induction allows me to conclude that $P(n)$ is true for all $n \geq 1$.

Towers of Hanoi ??



Ther minimum number of moves to move one tower onto another pin is $2^n - 1$

Definition: $T(n)$: the number of moves.

$B(n) = 2^n - 1$

$P(n)$: $T(n) = 2^n - 1$ for all $n \geq 1$

Proof by induction.

Basis step: is $P(1)$ true?

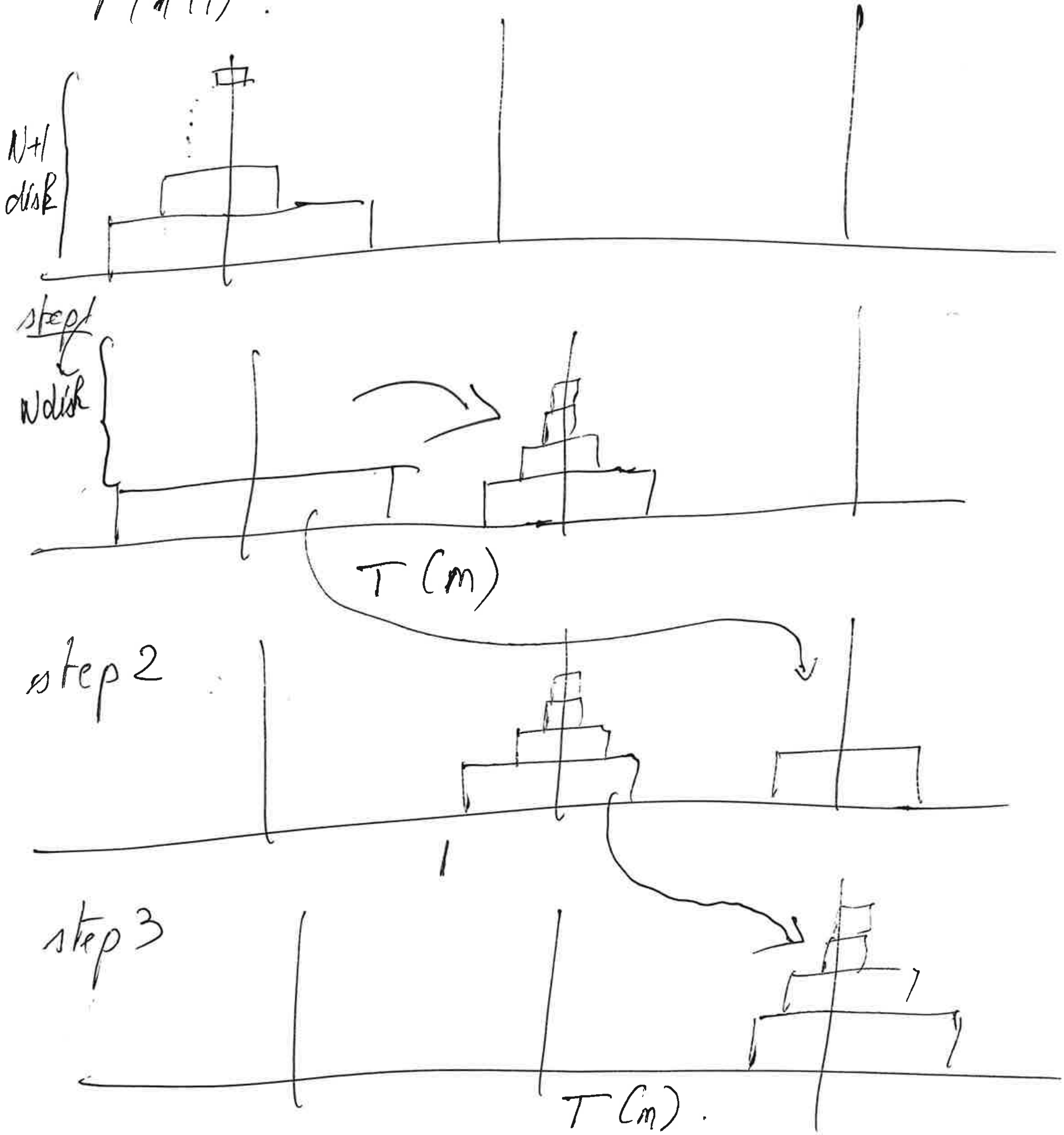
$T(1) = 1$ $\quad$ $T(1) = B(1)$, $P(1)$ is true

$B(1) = 2 - 1 = 1$

Inductive step: $P(n) \to P(n+1)$ is true for all $n \geq 1$

We assume $P(n)$ is true.

$P(n+1)$?



Total number of moves:

$$T(n+1) = 2\,T(n) + 1$$
$$= 2\,B(n) + 1$$
$$= 2\,(2^n - 1) + 1$$
$$= 2^{n+1} - 1$$

$$B(n+1) = 2^{n+1} - 1$$

Therefore $T(n+1) = B(n+1)$: $P(n+1)$ is true.

The method of proof by induction allows me to conclude that $P(n)$ is true for all $n \geq 1$.