Counting

I) Set theory

Definition: A set is an unordered collection of objects.

Example of sets:

- Days of the week
- Students in ECO17
- $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$

Terminology:

$\emptyset$: Empty set: does not contain any ~~num~~ objects.

To write that an object x belongs to a set A, we use the symbol $\in$

$x \in A$ means x belongs to A.

To write that a set A is fully included into a set B, we use the symbol $\subset$

$$A \subset B$$

How do we describe sets?

The set S of natural numbers that are smaller of equal to 10:

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

Set of even numbers that are natural and smaller or equal to 20

$$S = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

These representations are called roster representations.

Set building representation of a set:

Set of natural numbers that are smaller or equal to 10,000:

$$S = \{ x \in \mathbb{N} \mid x \leq 10{,}000 \}$$

$$A = \{ x \in \mathbb{N} \mid 2 \leq x \leq 5 \}$$
$$= \{ 2, 3, 4, 5 \}$$

More definitions

a) Union: the union of a set A and a set B, denoted $A \cup B$, is the set that contains those elements that are either in A, or in B or in both.

$$A \cup B = \{ x \mid x \in A \vee x \in B \}$$

Intersection

The intersection of two sets A and B, denoted $A \cap B$, is the set that contains those elements that are ~~either~~ in A and in B

$$A \cap B = \{ x \mid x \in A \wedge x \in B \}$$

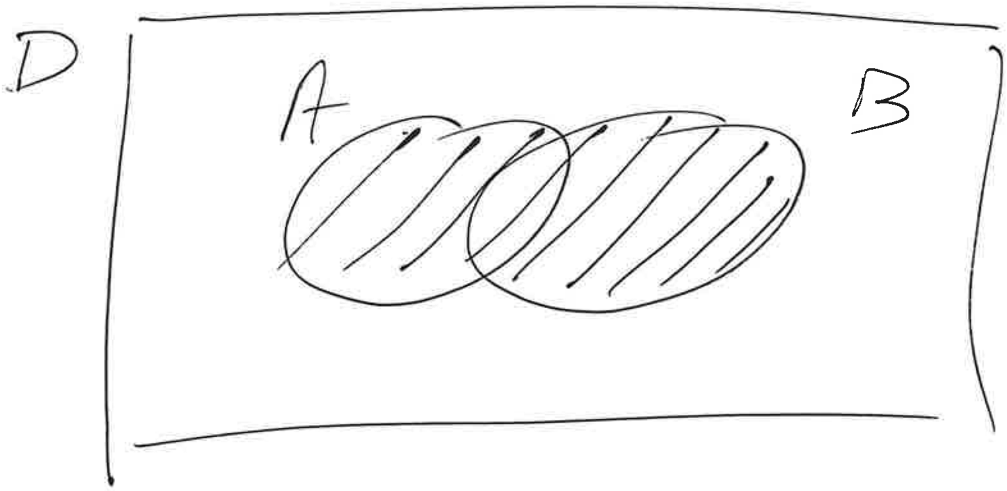
Complement

Given a domain D and a set A that belongs to D, the complement of A in D is the set of elements of D that do not belong to A

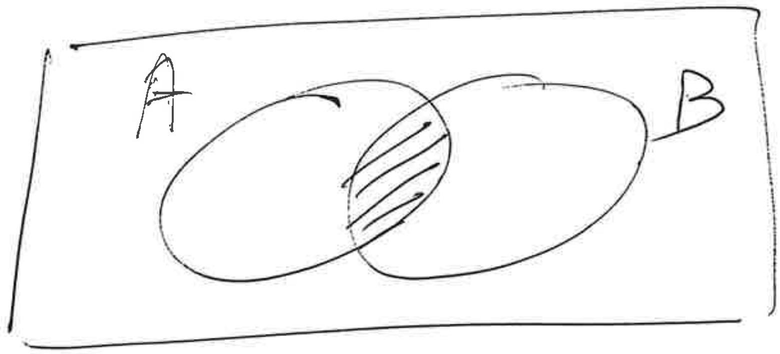
$$\overline{A} = \{ x \in D \mid x \notin A \}$$

$$= \{ x \in D \mid \neg (x \in A) \}$$

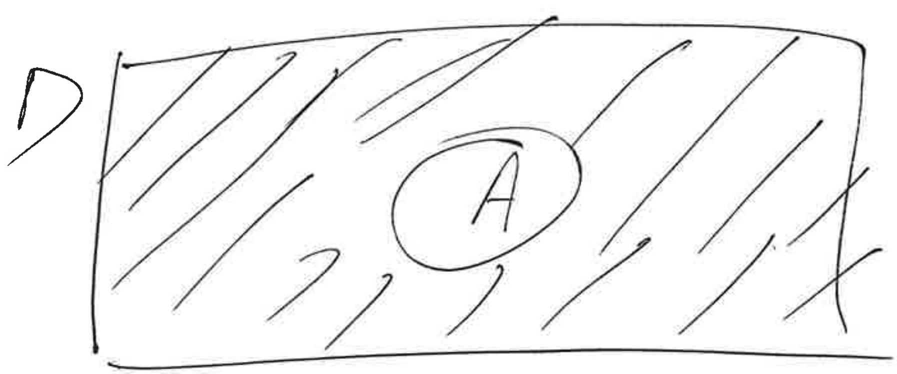
Visualization



$A \cup B$



$A \cap B$



$\overline{A}$

Remember

$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

Other properties.

For a set A in a domain D,

$$A \cup \overline{A} = D$$

$$A \cap \overline{A} = \emptyset$$