Counting 3/7

I) Set theory

Set is an unordered list of objects.

A set is defined according to a domain.

Terminology:

$x \in A$ (belongs)

$x \notin A$ (does not belong)

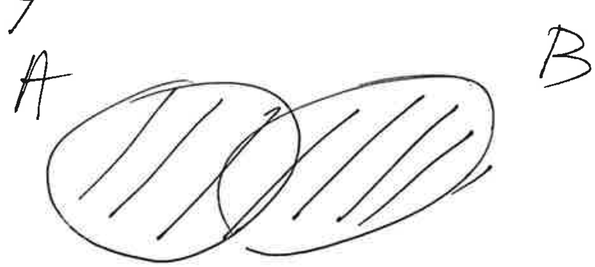
inclusion

$A \subset B$

Empty set: $\emptyset$

$\emptyset \subset A$

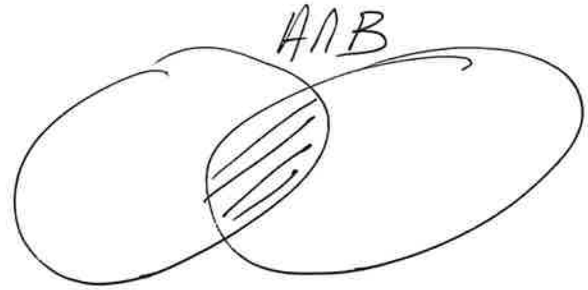
Union: $A \cup B$



$$A \cup B = \{ x \mid x \in A \vee x \in B \}$$

Intersection

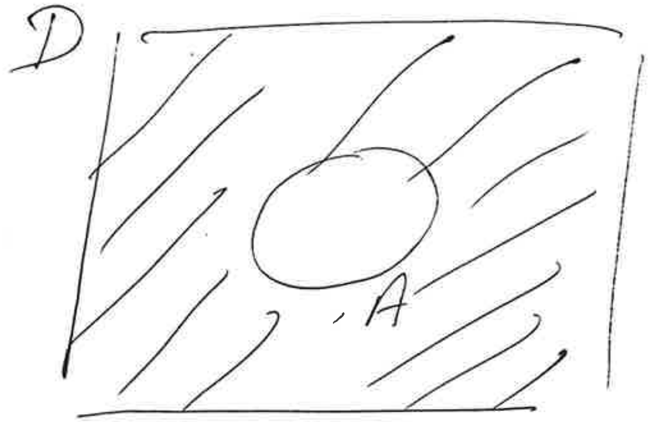
$$A \cap B$$



$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

Complement:

$$\overline{A}$$



$$\overline{A} = \{x \mid x \notin A\}$$

$$= \{x \mid \neg(x \in A)\}$$

Properties:

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

$$\overline{A \cup B} = \{x \,/\, x \notin A \cup B\}$$

$$= \{x \,/\, \neg (x \in A \cup B)\}$$

$$= \{x \,/\, \neg (x \in A \vee x \in B)\}$$

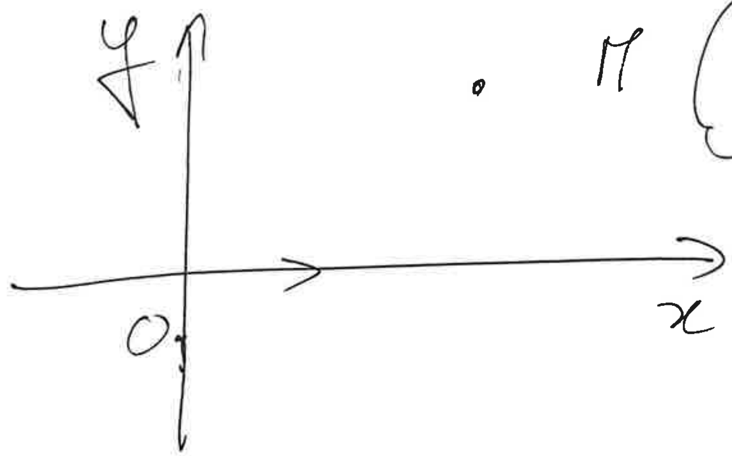
$$= \{x \,/\, \neg (x \in A) \wedge \neg (x \in B)\}$$

$$= \{x \,/\, x \in \overline{A} \wedge x \in \overline{B}\}$$

$$= \overline{A} \cap \overline{B}$$

Cartesian product of two sets A and B:

The cartesian product of two sets A and B is the list of ordered pair of objects (a, b) such that $a \in A$ and $b \in B$.



$\cdot\; M(x, y) \in \underbrace{\mathbb{R} \times \mathbb{R}}_{\mathbb{R}^2}$

Counting

Counting is the concept of finding the number of element in a set.

Definition: The number of elements in a set A is called the cardinality of A, and written $|A|$

Properties

Cardinality of a Cartesian product

If you have 2 sets A and B whose cardinalities are $|A|$ and $|B|$

then the cardinality of the Cartesian product $A \times B$ is

$$|A \times B| = |A| \times |B|$$

<u>Extension:</u>

$$|A_1 \times A_2 \times \cdots A_n| = |A_1| \times |A_2| \cdots |A_n|$$

This defines the product rule in counting.

<u>Example:</u> $L = \{\text{knight, knave, spy}\}$

Island of knights, knaves, and spies. On that island, you meet 4 peoples, A, B, C, D.

When you meet one group:

$$A, \quad B, \quad C, \quad D$$

$$\downarrow$$

$$L \times L \times L \times L$$

$$|L \times L \times L \times L| = |L| \times |L| \times |L| \times |L|$$
$$= 3 \times 3 \times 3 \times 3$$
$$= 81$$

How many bit strings of length 8 can you form?

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$
$$= 2^8 = 256$$

Second property: inclusion - exclusion principle.

$$\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$$

If a set A belongs to a domain D then

$$|\overline{A}| = |D| - |A|$$