

## The ECS17 Prayers

- 1) Thou shalt not say "there exists  $k$ " without mentioning the domain of  $k$ .
- 2) Thou shalt not say "it is obvious"
- 3) If  $p$  and  $q$  are two propositions, then  $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$ . This is the basis for the proof by contrapositive.
- 4) If  $p$  and  $q$  are two propositions, then  $p \rightarrow q \Leftrightarrow \neg p \vee q$ . This is the basis for the proof by contradiction.
- 5) An integer  $n$  is even if and only if there exists an integer  $k$  such that  $n = 2k$ . We say also that  $n$  is a multiple of 2.
- 6) An integer  $n$  is odd if and only if there exists an integer  $k$  such that  $n = 2k + 1$ .
- 7) BEWARE of divisions and square roots when you are working with integers.

## Proofs that you can use without proving them again

We can use the following results without having to validate them:

- 1) Let  $n$  be an integer. Then:
  - a) If  $n$  is even, then  $n+1$  and  $n-1$  are odd
  - b) if  $n$  is odd, then  $n+1$  and  $n-1$  are even
- 2) Let  $n$  be an integer. Then:
  - a)  $n$  is even, if and only if  $n^2$  is even
  - b)  $n$  is odd, if and only if  $n^2$  is odd
- 3)  $\forall n \in \mathbb{Z}, n(n+1)$  is even.
- 4)  $\sqrt{2}$  is irrational.

## Identities

Let  $a$  and  $b$  be two real numbers:

- 1)  $(a+b)^2 = a^2 + 2ab + b^2$
- 2)  $(a-b)^2 = a^2 - 2ab + b^2$
- 3)  $a^2 - b^2 = (a-b)(a+b)$
- 4) **Completing the square:**  $a^2 + b^2 = (a+b)^2 - 2ab$