

The ECS17 Prayers

- 1) Thou shalt not say "there exists k " without mentioning the domain of k .
- 2) Thou shalt not say "it is obvious"
- 3) If p and q are two propositions, then $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$. This is the basis for the proof by contrapositive.
- 4) If p and q are two propositions, then $p \rightarrow q \Leftrightarrow \neg p \vee q$. This is the basis for the proof by contradiction.
- 5) An integer n is even if and only if there exists an integer k such that $n = 2k$. We say also that n is a multiple of 2.
- 6) An integer n is odd if and only if there exists an integer k such that $n = 2k + 1$.
- 7) BEWARE of divisions and square roots when you are working with integers.

Proofs that you can use without proving them again

We can use the following results without having to validate them:

- 1) Let n be an integer. Then:
 - a) If n is even, then $n+1$ and $n-1$ are odd
 - b) if n is odd, then $n+1$ and $n-1$ are even
- 2) Let n be an integer. Then:
 - a) n is even, if and only if n^2 is even
 - b) n is odd, if and only if n^2 is odd
- 3) $\forall n \in \mathbb{Z}, n(n + 1)$ is even.
- 4) $\sqrt{2}$ is irrational.

Identities

Let a and b be two real numbers:

- 1) $(a + b)^2 = a^2 + 2ab + b^2$
- 2) $(a - b)^2 = a^2 - 2ab + b^2$
- 3) $a^2 - b^2 = (a - b)(a + b)$
- 4) **Completing the square:** $a^2 + b^2 = (a + b)^2 - 2ab$