

**ECS20**  
**Discussion 2**  
**Week 2: 1/16-1/22, 2019**

**Exercise 1**

Let  $p$  and  $q$  be two statements.

- a) Write down the truth tables for  $(\neg p \vee q) \wedge q$  and  $(\neg p \wedge q) \vee q$ . What do you notice about the truth tables? Based on this result, a creative student concludes that you can always interchange the symbol “and” and “or” without changing the truth table. Is the student right?
- b) Write down the truth tables for  $(\neg p \vee q) \wedge p$  and  $(\neg p \wedge q) \vee p$ . What do you think of the rule formulated by the students in (a)?

**Exercise 2**

Let  $p$ ,  $q$ , and  $r$  be three propositions. Prove or disprove the following statements using truth tables:

- a)  $(p \vee \neg p) \wedge (q \vee r)$  is equivalent to  $q \vee r$
- b)  $(p \wedge \neg p) \vee (q \wedge r)$  is equivalent to  $q \wedge r$

Can you find a simpler way to prove the same properties?

**Exercise 3**

Construct a truth table for the compound propositions:

- a)  $p \oplus \neg q$
- b)  $\neg p \oplus \neg q$

**Exercise 4**

This exercise relate to the inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the way described. If you cannot determine what these two people are, can you draw any conclusions?

- a) A says “At least one of us is a knave” and B says nothing.
- b) A says “We are both knaves” and B says nothing.

**Exercise 5**

This exercise relates again to the inhabitants of the island created by Smullyan, except that this time there are three types of inhabitants, knights that always tell the truth, knaves that always lie, and spies that may tell the truth or lie. You encounter three people, A, B,

and C and you know that one is a knight, one is a knave, and one is a spy. Determine, if possible, what A, B, and C are if they address you in the way described. If you cannot determine what these three people are, can you draw any conclusions?

PA: A says ``C is a knave"

PB: B says ``A is the knight"

PC: C says ``I am the spy"