

ECS20
Discussion 3: January 23-28, 2019

Exercise 1

Let p and q be two propositions. The proposition p NOR q is true when both p and q are false, and it is false otherwise. It is denoted $p \downarrow q$

- a) Write down the truth table for $p \downarrow q$
- b) Show that $p \downarrow q$ is logically equivalent to $\neg(p \vee q)$
- c) Find a compound proposition logically equivalent to $p \rightarrow q$ using only the logical operator \downarrow

Exercise 2

Let $P(x)$ be the statement " $x=x^2$ ". If the domain consists of the integers, what are the truth values of the following statements:

- a) $P(0)$
- b) $P(1)$
- c) $P(2)$
- d) $P(-1)$
- e) $\exists x P(x)$
- f) $\forall x P(x)$

Exercise 3

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

- a) All dogs have fleas.
- b) There is a horse that can add.
- c) Every koala can climb.
- d) No monkey can speak French.
- e) There exists a pig that can swim and catch fish.

Exercise 4

- a) Let a and b be two real numbers. Prove that if $n = ab$, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$
- b) Prove or disprove that there is a rational number x and an irrational number y such that x^y is irrational.
- c) There exists no integer a and b such that $21a+30b = 1$