# Discussion 4: Solutions

ECS 20 (Winter 2019)

Patrice Koehl
koehl@cs.ucdavis.edu

January 30- February 5, 2019

## Exercise 1

We use a proof by membership.

Let $x \in B \cap C$; by definition of the intersection, $x \in B$ and $x \in C$. Since $x \in B$ and $B \subseteq A$, we conclude that $x \in A$. Therefore $B \cap C \subseteq A$.

Let $x \in B \cup C$; by definition of the union, $x \in B$ or $x \in C$. If $x \in B$ and $B \subseteq A$, we conclude that $x \in A$. If $x \in C$ and $C \subseteq A$, we conclude that $x \in A$. In all cases, $x \in A$. Therefore $B \cup C \subseteq A$.

## Exercise 2

- **a**. Let us define $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$. The $A - B = \{1, 2\}$ and $B - A = \{5, 6\}$.
- **b**. Let us define $A = \{1, 2, 3, 4\}$, $B = \{5, 6, 7\}$ and $C = \{3, 4, 8, 9\}$. Then $(A \cap B) \cup C = C = \{3, 4, 8, 9\}$ and $(A \cap C) \cup B = \{3, 4, 5, 6, 7\}$.

## Exercise 3

We use a membership table:

| $A$ | $B$ | $A - B$ | $B - A$ | $(A - B) \cup (B - A)$ | $A \cup B$ | $A \cap B$ | $(A \cup B) - (A \cap B)$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Column 5 and 8 are equal: the two sets are equal.

## Exercise 4

The only difficulty is to translate the problem in math.

We define $D$ the set of tiles (this is the "universe", or "domain"), $S$ the subset of squares, $T$ the set of triangles, $R$ the set of red tiles and $B$ the set of blue tiles.

We know:

- $|D| = 19$ (there are 19 tiles total)
- $|S| = 12$ (there are 12 squares)
- $|R| = 11$ (there are 11 red tiles)
- $|S \cap B| = 4$ (there are 4 blue squares)
- $S \cup T = D$ and $S \cap T = \emptyset$
- $R \cup B = D$ and $R \cap B = \emptyset$

We directly deduce:

- $|D| = |S| + |T|$, therefore $|T| = 7$
- $|D| = |R| + |B|$, therefore $|B| = 8$
- $S = (S \cap R) \cup (S \cap B)$ and $(S \cap R) \cap (S \cap B) = S \cap R \cap B = \emptyset$, therefore $|S \cap R| = |S| - |S \cap B| = 8$
- A) The number of tiles that are square or blue:

  First, we observe that there are 8 blue tiles. Then: $|S \cup B| = |S| + |B| - |S \cap B| = 12 + 8 - 4 = 16$
- B)The number of tiles that are triangles and red:

  $|T \cap R| = 3$
- C) The number of tiles that are red or squares: $|S \cup R| = |S| + |R| - |S \cap R| = 12 + 11 - 8 = 15$

## Exercise 5

Show that $\overline{A - B} = \overline{A} \cup B$.

We use a membership table:

| $A$ | $B$ | $A - B$ | $\overline{A - B}$ | $\overline{A}$ | $\overline{A} \cup B$ |
|---|---|---|---|---|---|
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 |

Column 4 and 6 are equal: the two sets are equal.