ECS20

Discussion 5: 2/6 to 2/13 2019

Exercise 0

Additional problems on proofs:

a) Let x and y be two integers. Show that if $2x+5y=14$ and $y \neq 2$, then $x \neq 2$.

b) Let x and y be two integers. Show that if x^2+y^2 is odd, then $x+y$ is odd

Exercise 1

Determine whether each of these functions is a bijection from R to R:

a) $f(x) = 2x+4$

b) $f(x) = x^2+1$

c) $f(x) = (x+1)/(x+2)$

d) $f(x)=(x^2+1)/(x^2+2)$

Exercise 2

Let S = {-1,0,2,4,7}. Find f(S) if:

a) $f(x) = 1$

b) $f(x) = 2x+1$

c) $f(x) = \left\lceil \frac{x}{5} \right\rceil$

d) $f(x) = \left\lfloor \frac{x^2+1}{3} \right\rfloor$

Exercise 3

Let S be a subset of a universe U. The characteristic function f_S of S is the function from U to the set {0,1} such that $f_S(x)=1$ if x belongs to S and $f_S(x) = 0$ if x does not belong to S. Let A and B be two sets. Show that for all x in U,

a) $f_{A \cap B}(x) = f_A(x) \cdot f_B(x)$

b) $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_A(x) \cdot f_B(x)$

Exercise 4

Let n be an integer. Show that $\left\lfloor \frac{n}{2} \right\rfloor \left\lceil \frac{n}{2} \right\rceil = \left\lfloor \frac{n^2}{4} \right\rfloor$