

Name: _____
ID: _____

ECS 20: Discrete Mathematics
Midterm
February 5, 2019

Notes:

- 1) Midterm is open book, open notes. No computers though...
- 2) You have 45 minutes, no more: We will strictly enforce this.
- 3) You can answer directly on these sheets (preferred), or on loose paper.
- 4) Please write your name at the top right of at least the first page that you turn in!
- 5) Please, check your work!

Part I: logic (2 questions; first 20 points (10 for a) and 10 for b)), second 10 points; total 30 points)

1) Let p , q , and r be three propositions. Using truth tables or logical equivalences, indicate which (if any) of the propositions below are tautologies, contradictions, or neither.

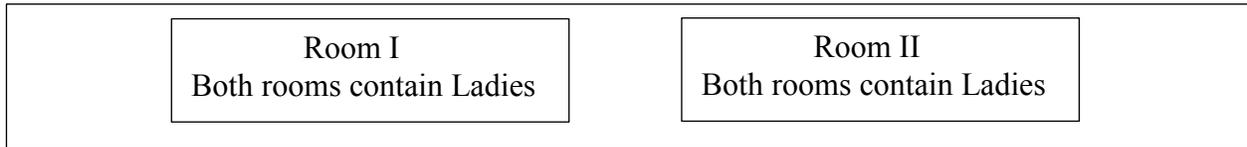
a) $(\neg p \vee q) \wedge [q \rightarrow (\neg r \wedge \neg p)] \wedge (p \vee r)$

Name: _____

ID: _____

b) $[(p \rightarrow q) \wedge (q \rightarrow \neg p)] \rightarrow p \leftrightarrow p$

2) Let us play a logical game. You find yourself in front of two rooms whose doors are closed. Behind each door, there could be a Lady or a Tiger. There is one sign on each door; you are told that if the first room contains a Lady, then the sign is true, but if a Tiger is in it, the sign is false. In room 2, the situation is the opposite: a Lady in room 2 means the sign is false, while a Tiger in room 2 means the sign is true. Here are the signs:



Can you say what is behind each door? Justify your answer.

Name: _____
ID: _____

Part II: proofs (3 questions; each 10 points; total 30 points)

Reminder:

- 0 is not a natural number
- An integer number n is odd if it can be written in the form $n = 2q + 1$, where q is an integer number
- An integer n is even if it can be written in the form $n = 2q$, where q is an integer

1) Let m and n be two integers. Show that if $m > 0$ and $n \leq -2$, then $m^2 + mn + n^2 \geq 0$

Name: _____

ID: _____

2) a) Let x be a real number, with $x \neq 0$. Use a proof by contradiction to show that if $x + \frac{1}{x} < 2$, then $x < 0$.

b) Repeat exercise 2)a, but this time using a direct proof.

Name: _____
ID: _____

3) Show that $2^{\frac{1}{4}}$ is irrational.

Part III: extra credit (5 points)

Let a and b be two integers. Use a direct proof to show that if $a^2 + b^2$ is even, then $a+b$ is even