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ECS 20: Discrete Mathematics
Midterm
February 5, 2019

Notes:

1) Midterm is open book, open notes. No computers though…
2) You have 45 minutes, no more: We will strictly enforce this.
3) You can answer directly on these sheets (preferred), or on loose paper.
4) Please write your name at the top right of at least the first page that you turn in!
5) Please, check your work!

Part I: logic (2 questions; first 20 points (10 for a) and 10 for b)), second 10 points; total 30 points)

1) Let *p, q,* and *r* be three propositions. Using truth tables or logical equivalences, indicate which (if any) of the propositions below are tautologies, contradictions, or neither.

a) $(\neg p \vee q) \wedge [q \rightarrow (\neg r \wedge \neg p)] \wedge (p \vee r)$

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b) $\left[[(p \rightarrow q) \wedge (q \rightarrow \neg p)] \rightarrow p\right] \leftrightarrow p$

2) Let us play a logical game. You find yourself in front of two rooms whose doors are closed. Behind each door, there could be a Lady or a Tiger. There is one sign on each door; you are told that if the first room contains a Lady, then the sign is true, but if a Tiger is in it, the sign is false. In room 2, the situation is the opposite: a Lady in room 2 means the sign is false, while a Tiger in room 2 means the sign is true. Here are the signs:

Room I	Room II
Both rooms contain Ladies	Both rooms contain Ladies

Can you say what is behind each door? Justify your answer.

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Part II: proofs (3 questions; each 10 points; total 30 points)

Reminder:

- 0 is not a natural number
- An integer number n is odd if it can be written in the form $n = 2q + 1$, where q is an integer number
- An integer n is even if it can be written in the form $n = 2q$, where q is an integer

1) Let m and n be two integers. Show that if $m > 0$ and $n \leq -2$, then $m^2 + mn + n^2 \geq 0$

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2) a) Let x be a real number, with $x \neq 0$. Use a proof by contradiction to show that if $x + \frac{1}{x} < 2$, then $x < 0$.

b) Repeat exercise 2)a, but this time using a direct proof.

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3) Show that $2^{\frac{1}{4}}$ is irrational.

Part III: extra credit (5 points)

Let *a* and *b* be two integers. Use a direct proof to show that if $a^2 + b^2$ is even, then $a+b$ is even