ECS 20: Discrete Mathematics
Midterm 1
October 28, 2014

Notes:
1) quizz is open book, open notes. No computers though…
2) You have 50 minutes, no more: I will strictly enforce this.
3) You can answer directly on these sheets (preferred), or on loose paper.
4) Please write your name at the top right of each page you turn in!
5) Please, check your work!
6) There are 8 questions, for a total of 100 points, and one extra credit problem, valued 5 points.

Part I: logic (2 questions, each 10 points; total 20 points)

Using truth tables, establish for each of the two propositions below if it is a tautology, a contradiction or neither

1) $(p \leftrightarrow q) \leftrightarrow (\neg p \leftrightarrow \neg q)$

2) $\left(p \rightarrow (q \wedge r)\right) \vee \left((p \wedge q) \rightarrow r\right)$

Part II: Logic satisfiability (two questions, each 15 points; total: 30).

1) A set of compound propositions is said to be satisfiable is we can assign truth-values to its variables that make each proposition true. For example, the set $\{p \wedge q, p \vee q\}$ is satisfiable as when we assign p and q to be **True**, both $p \wedge q$ and $p \vee q$ are true.

Prove, or disprove that the set $\{p \vee q, p \vee \neg q, \neg p \vee q, \neg p \vee \neg q \vee \neg r\}$ is satisfiable. If it is, give the possible truth-values for p, q, and r.

2) Smullyan's island

A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You meet three inhabitants: Alex, John and Sally. Alex says, `At least one of the following is true: that Sally is a knave or that I am a knight.' John says, `Alex could claim that I am a knave.' Sally claims, `Neither Alex nor John are knights.' Can you find who is a knight and who is a knave?

Part III: proofs (5 questions, each 10 points; total 50 points)

1) Prove that if $5n^2+2$ is even then n is even, where n is a natural number.

2) Let $a, b,$ and c be consecutive integers with $a<b<c$. Show that if $a \neq -1$ and $a \neq 3$, then $a^2 + b^2 \neq c^2$

3) *Show that* $\forall x \in Z, \quad \exists y \in Z, \quad 2y = x + x^2$

4) Show that for all n integer such that $1 \leq n \leq 5$, $n^2 - n + 11$ is prime

5) Show that every odd integer can be written as the difference between two perfect squares.

Extra credit (5 points)

You arrive in a country called Transylvania whose inhabitants are humans and vampires. Humans always tell the truth, while vampires always lie. However, both humans and vampires can be sane or insane. If an inhabitant is insane, she will believe that a truth statement is false, and a false statement is true. Sane inhabitants believe that truth statements are true and false statements are false. Thus sane humans and insane vampires make only true statements, while insane humans and sane vampires make only false statements. You meet two inhabitants, A and B. You know that one of them is a human, and the other is a vampire. A tells you: "we are both insane", while B tells you that "at least one of us is sane". From this, can you find which one is the vampire?