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# ECS 20: Discrete Mathematics
# Midterm
# November 14, 2016

***Notes:***

1) Midterm is open book, open notes. No computers though…
2) You have 40 minutes, no more: We will strictly enforce this.
3) You can answer directly on these sheets (preferred), or on loose paper.
4) Please write your name at the top right of at least the first page that you turn in!
5) Please, check your work!

**Part I: sets (1 question, 10 points)**

1) Let $A$ and $B$ be two sets in a domain $D$. Show that $\overline{\left(A \cap \overline{B}\right) \cup \left(B \cap \overline{A}\right)} = \left(\overline{A} \cap \overline{B}\right) \cup \left(B \cap A\right)$

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**Part II: functions (3 questions; each 10 points; total 30 points)**

1) Let $x$ be a ***real number***. Solve $\lfloor 3x-2 \rfloor = x$.

2) Let $x$ be a ***real number***. Show that $\left\lfloor \frac{x}{2} \right\rfloor + \left\lfloor \frac{x+1}{2} \right\rfloor = \lfloor x \rfloor$

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**Part III: Number theory (2 questions; each 10 points; total 20 points)**

1) Let *a, b, c* be three natural numbers. Show that if *b /a* and *c/a* and *gcd(b,c) = 1* then *(bc) / a*.

2) Show that there are no integer solutions to the equation $x^2-3y^2 = -1$

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3) Show that *13* divides $3^{126} + 5^{126}$.

**Part IV: extra credit (5 points)**

Let *x* be a real number. Find all positive non-zero solutions of $x\lfloor x \rfloor = x^2 - \lfloor x \rfloor^2$ where $\lfloor x \rfloor$ is the floor function.