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ECS 20: Discrete Mathematics
Midterm 2
November 20, 2014

Notes:

1) The midterm is open book, open notes.
2) You have 60 minutes, no more: I will strictly enforce this.
3) You can answer directly on these sheets (preferred), or on loose paper.
4) Please write your name at the top right of each page you turn in!
5) Please, check your work!
6) There are 7 questions (total of 100 points), and one extra credit problem, valued 5 points.

Part I: logic (2 questions, each 10 points; total 20 points)

Using truth tables, establish for each of the two propositions below if it is a tautology, a contradiction or neither

1) $\neg(\neg(p \to q)) \leftrightarrow (p \leftrightarrow q)$

2) $[p \leftrightarrow (\neg q \wedge \neg r)] \to [(\neg(q \wedge r)) \to p]$

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Part II: Logic puzzles (two questions, each 15 points; total: 30).

1) **Smullyan's island**. A very special island is inhabited only by knights and knaves. Knights always tell the truth, and knaves always lie. You meet three inhabitants: Alex, John and Sally. You ask Alex the question, "Are you a knight or a knave?" Alex answers, but rather indistinctly, so you cannot make out what he said. You ask then John, "What did Alex say?" John replies, "Alex said that he is a knave." At this point Sally says, "Don't believe John; he is lying!"
The question is, what are John and Sally? What about Alex?

2) Who robbed the National Bank? Inspector Malone knows that the culprit is one and only one of the following: Alex, John, or Sally. He interrogates them. Each makes two statements.
Alex: "It wasn't me." "John did it."
John: "Listen, Alex did it." "And Sally did it."
Sally: "I didn't do it." "Neither did Alex."
Each has made one true statement and one false statement. Who did it?

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Part III: proofs (4 questions, each 12.5 points; total 50 points)

1) Prove or disprove that $\forall n > 1$, where n is an integer, there are no 3 integers x, y and z such that $x^n + y^n = z^n$

2) Let A and B be two sets in a universe U. Show that $(A \cap B) \cup \overline{(A \cup \overline{B})} = B$

3) Let f be the function from Z to Z defined as, $\forall n \in Z$ $f(n) = n^2+4n+1$. Show that $\forall n \in Z$, if $f(n)$ is odd then $f(n+1)$ is even.

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4) Let *A, B*, and *C* be three sets in a universe *U*. Show that if they satisfy simultaneously: $A \bigcup B = C$, $(A \bigcup C) \bigcap B = C$, and $(A \bigcap C) \bigcup B = A$, then they are the same (i.e. A=B=C).

Extra credit (5 points)

Every inhabitant of the island of Bahava is either a knight who always tells the truth, a knave who always lies, or a spy who sometimes tells the truth and sometimes lies. Knights, knaves, and spies can be men or women. An old tradition on the island is that knights only marry knaves and knaves only marry knights. Hence a spy can marry only a spy. This problem is about two married couples, Mr. and Mrs. A, Mr. and Mrs. B. They are interviewed and three of the four people say:

Mr. A: Mr. B is a knight.

Mrs. A: My husband is right: Mr. B is a knight.

Mrs. B: That is right. My husband is indeed a knight.

What are each of the four people?