# Homework 2: Solutions ECS 20 (Winter 2019)

January 2, 2019

## Exercise 1

*Construct a truth table for each of these compound propositions:*

a) $A = (p \vee q) \rightarrow (p \oplus q)$

| $p$ | $q$ | $p \vee q$ | $p \oplus q$ | $A$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | T | T | T |
| F | T | T | T | T |
| F | F | F | F | T |

b) $A = (p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$

| $p$ | $q$ | $r$ | $p \leftrightarrow q$ | $\neg p$ | $\neg r$ | $\neg p \leftrightarrow \neg r$ | $A$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | F | F | T | F |
| T | T | F | T | F | T | F | T |
| T | F | T | F | F | F | T | T |
| T | F | F | F | F | T | F | F |
| F | T | T | F | T | F | F | F |
| F | T | F | F | T | T | T | T |
| F | F | T | T | T | F | F | T |
| F | F | F | T | T | T | T | F |

c) $(A = p \oplus q) \rightarrow (p \oplus \neg q)$

| $p$ | $q$ | $\neg q$ | $p \oplus q$ | $p \oplus \neg q$ | $A$ |
|---|---|---|---|---|---|
| T | T | F | F | T | T |
| T | F | T | T | F | F |
| F | T | F | T | F | F |
| F | F | T | F | T | T |

## Exercise 2

*Construct a truth table for each of these compound propositions:*

a) $A = (\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

| $p$ | $q$ | $r$ | $\neg p$ | $\neg q$ | $\neg p \leftrightarrow \neg q$ | $q \leftrightarrow r$ | $A$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | T | T | T |
| T | T | F | F | F | T | F | F |
| T | F | T | F | T | F | F | T |
| T | F | F | F | T | F | T | F |
| F | T | T | T | F | F | T | F |
| F | T | F | T | F | F | F | T |
| F | F | T | T | T | T | F | F |
| F | F | F | T | T | T | T | T |

b) $(p \oplus q) \wedge (p \oplus \neg q)$

| $p$ | $q$ | $\neg q$ | $p \oplus q$ | $(p \oplus \neg q)$ | $(p \oplus q) \wedge (p \oplus \neg q)$ |
|---|---|---|---|---|---|
| T | T | F | F | T | F |
| T | F | T | T | F | F |
| F | T | F | T | F | F |
| F | F | T | F | T | F |

## Exercise 3

*Show that the following is a tautology:* $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$.

Let us define $A = (p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$. The proposition is $A \rightarrow r$.

| $p$ | $q$ | $r$ | $p \vee q$ | $p \rightarrow r$ | $q \rightarrow r$ | $A$ | $A \rightarrow r$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | T | T | T | T | T |
| T | F | F | T | F | T | F | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | T | F | F | T |
| F | F | T | F | T | T | F | T |
| F | F | F | F | T | T | F | T |

## Exercise 4

*The Fair Maiden Rowena wishes to wed. And her father, the Evil King Berman, has devised a way to drive off suitors. He has a little quiz for them, and here it is. It's very simple:*

Three boxes sit on a table. The first is made of gold, the second is made of silver, and the third is made of lead. Inside one of these boxes is a picture of the fair Rowena. It is the job of the White Knight to figure out, without opening them, which one has her picture.

Now, to assist him in this endeavor there is an inscription on each of the boxes. The gold box says,"Rowena's picture is in this box." The silver box says, "The picture is not in this box." The lead box says, "The picture is not in the gold box." Only one of the statements is true. Which box holds the picture?

The simplest approach to solve this problem is to check systematically if the Gold box, the Silver box, or the Lead box contains the picture. In each case, we test the validities of the three statements.

| Box with picture | Golden Box<br>picture is in this box | Silver Box<br>picture is not in this box | Lead Box<br>picture is not in Gold box |
|---|---|---|---|
| Gold | True | True | False |
| Silver | False | False | True |
| Lead | False | True | True |

If the prize were in the Gold or Lead box, two of the propositions would be true, whereas if the prize is in Silver Box, only one proposition would be true. The latter is therefore true, and the prize is in the Silver Box.

## Exercise 5

*This exercise relate to the inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the way described. If you cannot determine what these two people are, can you draw any conclusions?*

A says "The two of us are both knaves", and B says "A is a knave".

We proceed as in class. We check all possible "values" for A and B, as well as the veracity of their statements:

We can eliminate:

- Line 1, as B would be a knight but he lies
- Line 2, as A would be a knight but he lies
- Line 4 as B would be a knave but he says the true

Line 3 is valid, and it is the only one. Therefore, A is a knave and B is a knight.

| Line number | A | B | A says "The two of us are both knaves" | B says "A is a knave" |
|---|---|---|---|---|
| 1 | Knight | Knight | F | F |
| 2 | Knight | Knave | F | F |
| 3 | Knave | Knight | F | T |
| 4 | Knave | Knave | T | T |

## Exercise 6

a) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

We need to show that the two propositions $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ have the same truth values:

| $p$ | $q$ | $r$ | $q \vee r$ | $p \wedge q$ | $p \wedge r$ | $p \wedge (q \vee r)$ | $(p \wedge q) \vee (p \wedge r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | F | T | T |
| T | F | T | T | F | T | T | T |
| T | F | F | F | F | F | F | F |
| F | T | T | T | F | F | F | F |
| F | T | F | T | F | F | F | F |
| F | F | T | T | F | F | F | F |
| F | F | F | F | F | F | F | F |

b) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

We need to show that the two propositions $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ have the same truth values:

| $p$ | $q$ | $r$ | $q \wedge r$ | $p \vee q$ | $p \vee r$ | $p \vee (q \wedge r)$ | $(p \vee q) \wedge (p \vee r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | F | T | T | T | T |
| T | F | T | F | T | T | T | T |
| T | F | F | F | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | T | F | F | T | F | F | F |
| F | F | T | F | F | T | F | F |
| F | F | F | F | F | F | F | F |

## Exercise 7

*Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are equivalent. We show that these two statements have the same truth values:*

| $p$ | $q$ | $p \wedge q$ | $\neg p$ | $\neg q$ | $\neg p \wedge \neg q$ | $(p \wedge q) \vee (\neg p \wedge \neg q)$ | $p \leftrightarrow q$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | T | T |
| T | F | F | F | T | F | F | F |
| F | T | F | T | F | F | F | F |
| F | F | F | T | T | T | T | T |

## Exercise 8

*Use either a truth table or logical equivalence to show that $(p \to q) \wedge (p \to r) \Leftrightarrow p \to (q \wedge r)$*

We will use a table of truth and logical equivalence:

a) Table of truth

We show that the two statements $A = (p \to q) \wedge (p \to r)$ and $B = p \to (q \wedge r)$ have the same truth values:

| $p$ | $q$ | $r$ | $p \to q$ | $p \to r$ | $A$ | $q \wedge r$ | $B$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | F |
| T | F | T | F | T | F | F | F |
| T | F | F | F | F | F | F | F |
| F | T | T | T | T | T | T | T |
| F | T | F | T | T | T | F | T |
| F | F | T | T | T | T | F | T |
| F | F | F | T | T | T | F | T |

b) We use logical equivalence.

$$
\begin{array}{ccl|c}
A & \Leftrightarrow & (p \to q) \wedge (p \to r) & \text{Problem definition} \\
 & \Leftrightarrow & (\neg p \vee q) \wedge (\neg p \vee r) & \text{property of implication} \\
 & \Leftrightarrow & \neg p \vee (q \wedge r) & \text{Distributivity} \\
 & \Leftrightarrow & p \to (q \wedge r) & \text{property of implication} \\
 & \Leftrightarrow & B &
\end{array}
$$

## Extra Credit

*We are back on the island of knights and knaves (see exercise above). John and Bill are residents. John: if Bill is a knave, then I am a knight Bill: we are different Who is who?*

We proceed as for exercise 5: we check all possible "values" for John and Bill, as well as the veracity of their statements. Note that John's statement is an implication.

| Line number | John | Bill | John says "If Bill is a knave, then I am a knight" | Bill says "We are different" |
|---|---|---|---|---|
| 1 | Knight | Knight | T | F |
| 2 | Knight | Knave | T | T |
| 3 | Knave | Knight | T | T |
| 4 | Knave | Knave | F | F |

We can eliminate:

- Line 1, as Bill would be a knight but he lies
- Line 2, as Bill would be a knave but he tells the truth
- Line 3 as John would be a knave but he says the true

Line 4 is valid, and it is the only one. Therefore, both John and Bill are knaves.