Homework 7: due 2/26/2019

ECS 20 (Winter 2019)

Patrice Koehl
koehl@cs.ucdavis.edu

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Exercise 1: 10 points

a) Let a be a natural number strictly greater than 1. Show that $\gcd(a, a-1) = 1$.

b) Use the result of part a) to solve the Diophantine equation $a + 3b = ab$ where a and b are two positive integers.

Exercise 2: 20 points (10 for a), 10 for b))

a) Let a, b, and c be three integers. Show that the equation $ax + by = c$ has at least one solution in $\mathbb{Z}^2$ if and only if $\gcd(a, b)/c$.

b) A group of men and women spent \$100 in a store. Knowing that each man spent \$7, and each woman spent \$6, can you find how many men and how many women are in the group?

Exercise 3: 20 points (10 for a), 10 for b))

a) Let a and b be two natural numbers. Show that if $\gcd(a, b) = 1$ then $\gcd(a, b^2) = 1$.

b) Let a and b be two natural numbers. Show that if $\gcd(a, b) = 1$ then $\gcd(a^2, b^2) = 1$.

Exercise 4: 10 points

Let n be a natural number such that the remainder of the division of 5218 by n is 10, and the remainder of the division of 2543 by n is 11. What is n?

Exercise 5: 10 points

Find all $(x, y) \in \mathbb{N}^2$ that satisfy the system of equations:

$$\begin{cases} x^2 - y^2 = 2340 \\ \gcd(x, y) = 6 \end{cases}$$

Exercise 6: 10 points

Let n be a natural number. We define $A = n - 2$ and $B = n^2 - 6n + 13$. Show that $\gcd(A, B) = \gcd(A, 5)$.

Exercise 7: 10 points

Let a and b be two natural numbers. Solve the equations $a^2 - b^2 = 13$.

Extra Credit: 5 points

Let a and b be two natural numbers. Solve $\gcd(a, b) + \text{lcm}(a, b) = b + 9$.